Climate Risk Pricing

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Abstract

We develop an environmental macro-finance model to study how markets price transition and physical climate risks. When carbon taxes are below the social cost of carbon, raising them improves long-run welfare even as they reduce current output. Brown firms perform well in the "bad economic states" when low taxes let climate damage worsen. This hedging value gives them lower required returns, reversing standard ESG predictions. The green-minus-brown required return can, however, switch sign depending on how policy and climate shocks interact. Strikingly, climate-concerned investors optimally hedge by holding brown stocks, while skeptics hedge with green. More broadly, private hedging motives can diverge from societal climate objectives.

Keywords: climate risk, ESG investing, sustainable finance, greenium

JEL Codes: E44, G12, H23, Q5

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Climate risk affects firms both through carbon taxes (transition risk) and through the economic damages of a warming planet (physical risk). Both sources of risk shape firms' cash flows, investors' portfolios, and equilibrium asset prices. A fast-growing empirical literature shows that green stocks tend to outperform when climate concerns rise (Engle et al., 2020; Krueger et al., 2020; Bolton and Kacperczyk, 2021, 2023; Ardia et al., 2023; Faccini et al., 2023; Sautner et al., 2023) and similarly for corporate bonds (Huynh and Xia, 2021; Seltzer et al., 2022). This literature, along with many financial regulators and practitioners, argues that investors should hedge climate risk by buying green assets and shorting brown ones, thus lowering the cost of capital for green assets.[1] Yet, this conventional wisdom lacks a theoretical foundation: *do* green assets hedge climate risk?

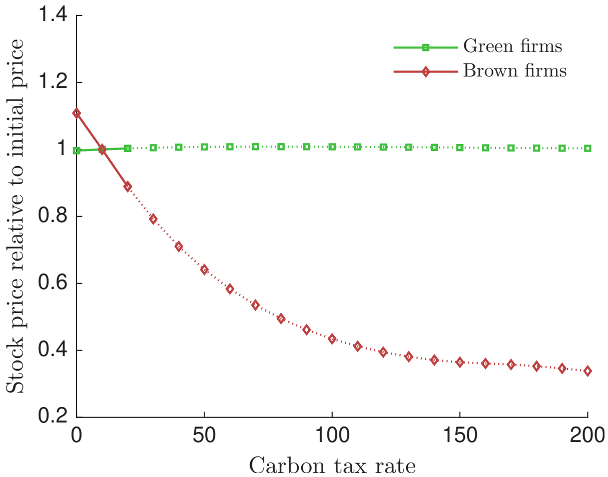
We develop a tractable environmental macro-finance general-equilibrium model to answer this question. The model delivers a surprising result: brown assets can serve as hedges, and green assets can carry systematic risk. Hence, risk pricing can imply lower required returns to brown assets relative to green, which perversely affects firms' incentives for a green transition.

Intuition. The conventional wisdom is that brown stocks face the risk of rising carbon taxes, so their required returns should be higher. Excess required returns, however, are given by the covariance between realized returns and the stochastic discount factor (SDF), M, which reflects the representative agent's marginal utility: $E(r^e) = -cov(r^e, M)/E(M)$. Hence, to understand climate risk pricing, consider how carbon-tax shocks jointly affect returns and the SDF.

Clearly, increases in carbon taxes predominantly hurt brown firms, so their stock prices drop relative to green ones, consistent with the findings of the above-cited papers. Increases in carbon taxes also lower the SDF: The key observation is that welfare is maximized when carbon taxes equal the social cost of carbon. Hence, when carbon taxes are sub-optimally

[1]The argument in the academic literature is articulated, for example, by Bolton and Kacperczyk (2021), is that "major curbs in CO_2 emissions are likely to be introduced over the next decade. Primarily affected by these curbs are the companies with operations generating high CO_2 emissions," so "one would expect to see the risk with respect to carbon emissions to be reflected in the cross-section of stock returns." Many of the world's financial supervisors and central banks have formed a network "to enhance the role of the financial system to manage risks and to mobilise mainstream finance in the context of environmentally sustainable development" (The Network for Greening the Financial System, NGFS). In a survey of financial professionals, Bauer et al. (2024) find that 67% believe that climate risks are extremely, very, or somewhat important for the pricing of stocks.

(a) **Brown returns decrease in carbon tax** (b) **SDF is U-shaped in carbon tax**



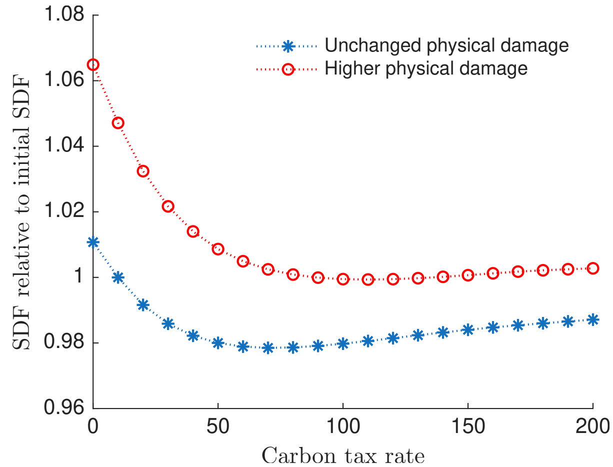


Figure 1: **Contemporaneous returns and SDF.** Panel (a) shows the contemporaneous returns of green and brown firms (y-axis) in connection with changes to the carbon tax (x-axis). Panel (b) shows the corresponding stochastic discount factor when physical risk in unchanged (blue asterisks), as well as the SDF for a higher level of environmental damage (red circles). Further details and parameters are given in Section 4.

low — current global carbon taxes are, indeed, well below most estimates of the social cost of carbon — an unexpected increase in carbon taxes raises welfare. Further, in a simple one-period setting in which production and consumption take place at the end, the higher welfare means higher consumption, and thus a lower SDF.

In sum, higher carbon taxes lead to both low brown stock returns and a low SDF. This joint behavior corresponds to positive covariance between returns and the SDF, which translates into low required returns.

Said differently, from a hedging perspective, the representative investor seeks protection against the adverse scenario in which carbon taxes fall and climate damages accelerate. This is the scenario of low aggregate welfare (high SDF), and it is brown stocks that fare relatively better when it occurs. Consequently, the greenium — the required return of green stocks relative to brown — is positive.

Level-growth tradeoff. While this intuition captures the basic mechanism, our dynamic model introduces an important intertemporal tradeoff between the level and growth of output. Indeed, in a dynamic economy, an increase in the carbon tax only gradually leads to an environmental benefit, but immediately lowers output as production is switched to greener firms. Hence, an increase in carbon taxes lowers current aggregate consumption, but raises future consumption growth by mitigating environmental damage.

SDF. In such a dynamic environment, how does marginal utility, that is, the SDF, depend on the carbon tax? The answer is not straightforward since a higher tax has the benefit of a higher growth rate, but the cost of a lower current consumption. Therefore, the SDF is U-shaped in the carbon tax, as seen in Figure 1(b), which displays the SDF for a range of carbon-tax values and two different levels of physical damages from emissions (explained further below). When the carbon tax is low (placing the economy on the left side of the U), then a small increase in the tax lowers the SDF (moving the economy toward the bottom of the U), corresponding to an improvement in welfare as in the intuitive two-period example.

Greenium driven by transition risk. Combining the results on contemporaneous returns and on the SDF seen in Figure 1, we find that green assets outperform brown assets when carbon taxes increase and the SDF is low (good state), and, conversely, brown assets outperform when the SDF is high (bad state). Therefore, brown assets have lower equilibrium expected returns, a positive greenium. More broadly, Figure 2 illustrates a calibration of how required returns depend on how firms' pollution as measured by their emission intensities. The downward-sloping line reflects the positive expected return of a green-minus-brown (GMB) portfolio. This surprising results relies on the following conditions.

First, the current carbon tax must be low enough that the economy is on the left side of the U from Figure 1(b). Indeed, required returns depend on the covariance between realized returns and the SDF and, as seen in Figure 1, the GMB portfolio has a negative correlation on the left side of the U. In contrast, when the carbon tax is high and the economy is on the right-hand side of the U, the correlation becomes positive, reversing the sign of the greenium.

Second, transition and physical risks must be weakly correlated. A strong co-movement between the two blurs the distinction between good and bad states, as discussed next.

Physical risk. We model physical risk through emission externalities, causing environmental damage that gradually erodes productivity. This damage includes both predictable and stochastic components, the latter constituting physical risk. Physical risk can be idiosyncratic or systematic. We focus on systematic physical risk, namely the risk that the

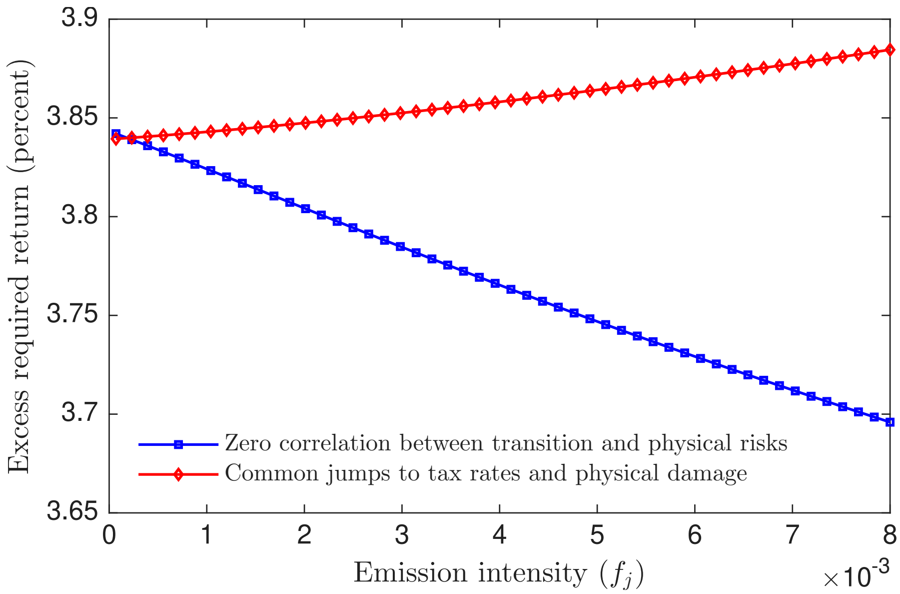


Figure 2: **Required return–emission relation with climate risk pricing.** This figure shows the required returns (y-axis) across firms that differ in their emission intensities (x-axis). The lower line (square markers) depicts this return-emission relation when transition risk and physical risk are uncorrelated. This downward-sloping line reflects that more polluting firms (on the right) have slightly lower required returns than cleaner firms (on the left) in this case. The upper line (diamond markers) depicts the relation when transition risk and physical risk are positively related such that carbon taxes increase when physical risk worsens. The upward-sloping line shows that the risk premium switches sign. Further details and parameters are given in Section 4.

total environmental damage becomes larger than expected. Idiosyncratic physical risk means that some firms are more affected than others, but such exposures need not be related to whether firms are green or brown.

Of course, a higher damage has a negative welfare impact and increases marginal utilities. This fact renders the effect of transition risk more nuanced, tying it to how it interacts with physical risk. If a higher physical risk leads to higher carbon taxes, then the SDF can be higher when carbon taxes rise. In other words, when taxes increase autonomously from a welfare-improving policy, the SDF falls; but if the same tax hike is triggered by a negative climate shock, the SDF rises because the shock worsens aggregate welfare. Figure 1(b) shows this clearly: the SDF under a physical climate shock (circle markers) is above the baseline SDF (marked with asterisks).

For this reason, we show that the required return-emission relation is less downward-sloping, or even upward-sloping, when physical and transition risks are more correlated. This results is illustrated in Figure 2, where the upper line has correlated physical and transition risks while the lower line has independent risks.

Hence, the overall SDF response depends on the joint distribution of policy and climate shocks, a relationship that must be quantified through calibration. The broader implication is that risk-based considerations differ fundamentally from moral or preference-based motives: individual hedging incentives do not necessarily align with societal goals for climate mitigation.

Bet on the other team. When investors hold heterogeneous beliefs about climate risk, who should hold more green versus brown assets? We show that investors concerned about climate risk optimally hedge by holding brown assets, which perform well when carbon taxes remain low and climate outcomes are adverse. In other words, climate concerned investors might seek to ensure that they can buy the higher ground if the world is about to flood.

In contrast, climate-skeptical investors hedge by buying green stocks, since these stocks outperform brown in a scenario of high carbon taxes, which skeptics view as a scenario of destructive over-regulation.

By analogy, sports fans who wish to hedge the risk that their team loses sometimes bet on the opposing team. Either your team wins, or you make some money.

Calibration. We quantify the model by calibrating it to observed emissions data and estimates of the social cost of carbon from Nordhaus (2019) and EPA (2023). We find that the greenium due to climate risk pricing is low in absolute magnitude as seen in Figure 2. We analyze the greenium across a range of specifications and find that the sign of the greenium can vary, but the magnitude is below one percentage point in all calibrations.

Literature. Our analysis relates to four strands of research analyzing, respectively, green returns, policy uncertainty, sustainable investing, and integrated assessment models. First, our results have implications for whether sustainable investment can address climate change. A key mechanism with which green investment is hoped to address climate change is by lowering the cost of capital of green firms, while raising the cost of capital for brown firms. Pedersen (2023) shows how the greenium translates into an equivalent carbon tax, thus showing how much required returns must change to drive a green transition. The greenium can arise simply out of investors' preferences for owning green assets (Pástor et al., 2021; Pedersen et al., 2021), and papers studying the greenium empirically include Pástor et al. (2022), Bolton and Kacperczyk (2021), Zhang (2023), and Eskildsen et al. (2024). Pástor et al. (2025a) find that firms with a low "carbon burden," which captures future carbon emissions, sell at a premium.

However, many investors are unwilling to sacrifice meaningful returns to hold greener assets, and institutional managers face fiduciary duties to maximize financial performance (Edmans et al., 2024). Therefore, regulators, industry advocates of sustainable investing, and the literature on climate risk cited in the first paragraph propose that a greenium can arise instead from risk considerations. Central banks have imposed climate stress tests (Acharya et al., 2023) and the Network for Greening the Financial System (2020), a group of many of the world's financial supervisors and central banks, emphasizes that financial institutions

> "need to accurately assess the climate and environmental risks to which they are exposed. Underestimating these risks leads to excessive allocation of financial resources to polluting or high carbon sectors."

In other words, these supervisors assume that correct risk pricing pushes capital to green assets, but our results caution that individual risk-hedging motives can induce demand for brown assets, reversing the sign of the greenium and working against collective climate objectives. Related work includes Baker et al. (2022), who consider productivity risk in a one-period model and find that green firms' productivity shocks carry a higher risk price. Giglio et al. (2021b) provide a reduced-form framework accounting for climate damage and economic activity, discussing that assets with low payoffs when climate damage is high may be riskier or less risky. Bansal et al. (2020, 2021) numerically analyze models in which physical climate change is a long-run risk that lowers aggregate equity prices. Giglio et al. (2021a) and Pástor et al. (2025b) survey this literature. We complement this literature by solving a dynamic general-equilibrium production economy with stochastic carbon-tax policy, deriving intuitive implications for climate risk pricing across assets. In contrast to these papers, which consider exogenous cash flows without a carbon tax, we endogenize firm and investor behavior in light of carbon taxes, yielding an endogenous risk premium jointly shaped by policy and physical risks.

Second, our model thus creates a connection between the literature on green returns and that on policy uncertainty (Baker et al., 2016; Kelly et al., 2016). Stroebel and Wurgler (2021) present survey evidence that policy risk is currently viewed as the top climate risk to businesses and investors, and we show how this risk is priced.

Third, our results are related to the literature that studies sustainable portfolio behavior across investors. Our "bet-on-the-other-team" result contrasts with conventional intuition, offering a normative benchmark rather than a positive prediction: there are likely few, if any, green investors who buy brown stocks as a hedge or anti-ESG investors who buy green stocks as a hedge. Most green investors buy green stocks either to align their investments with their values or to lower the cost of capital for green firms (Bonnefon et al., 2025). Some investors also claim to buy green stocks based on a risk argument similar to that in the existing literature (Giglio et al., 2025), but risk is often a minor argument, perhaps because of its tenuous foundation, as we show. Addressing climate change by hoping that investors will buy green assets as a hedge despite a tenuous theoretical foundation is unlikely to be a sustainable path toward net zero emission.

Fourth, our framework connects to integrated assessment models in environmental macroeconomics such as Nordhaus (1994), Peck and Teisberg (1992), Golosov et al. (2014), Daniel et al. (2019), Barnett et al. (2020, 2024), Chikhani and Renne (2025), and other papers surveyed in Nobel Committee (2018). Our model offers a tractable bridge between these integrated assessment models and macro-finance asset pricing. We also complement models featuring other welfare and political dimensions of sustainable finance (Hong et al., 2023a,b; Allen et al., 2023; Heeb et al., 2023).

In sum, we develop a novel model for pricing transition and physical risks. Its predictions for the greenium and investor portfolios challenge conventional wisdom and highlight how policy uncertainty can undermine climate policy goals.

1 Model

We consider a production economy populated by firms and a representative household who invests, works, and consumes at each time $t \in [0, \infty)$. The economy is modeled by extending a Dixit-Stiglitz framework such that firms' emissions affect future productivity, thereby providing a tractable representation of climate risk. We now describe each element of the model in detail.

1.1 Firm producing final goods

A competitive firm produces Y_t units of a final good, which are sold to households for consumption. The final good is produced using intermediate goods as inputs, denoted as Y_{it} for all $i \in [0, 1]$. The final-good production function is

$$Y_t = \left(\int_0^1 Y_{it}^{\alpha} di \right)^{\frac{1}{\alpha}}, \tag{1}$$

where $\alpha \in (0, 1)$. An α close to 1 means that intermediate goods are easily interchangeable, while a smaller α corresponds to more distinct goods.

The final good is used as numéraire, meaning that the final good is sold to households for the price of 1, while the endogenous price of each intermediate good is p_{it}. Thus, the operating profit, Π_t, of the final-good firm equals

$$\Pi_t = \left(\int_0^1 Y_{it}^{\alpha} di \right)^{\frac{1}{\alpha}} - \int_0^1 p_{it} Y_{it}\, di. \tag{2}$$

1.2 Firms producing intermediate goods

Each intermediate-good firm $i \in [0, 1]$ produces an output, Y_{it}, by employing L_{it} units of labor according to the production function

$$Y_{it} = A_t L_{it}, \tag{3}$$

where A_t is a productivity process, common to all firms.

Each firm also creates emission externalities X_i proportional to its output

$$X_{it} = f_i Y_{it}, \tag{4}$$

where f_i is the "fossil intensity" of the firm's technology.

Emissions are taxed at a rate τ_t, so that firm i pays a carbon tax of $\tau_t X_{it}$. The firm also has labor cost $\mathcal{W}_t L_{it}$, where $\mathcal{W}_t$ is the endogenous unit wage. Hence, each intermediate good firm realizes a profit, Π_{it}, at time t of

$$\Pi_{it} = p_{it} Y_{it} - \tau_t f_i Y_{it} - \mathcal{W}_t L_{it}. \tag{5}$$

Intermediate good producers are monopolistically competitive: They set their price, p_{it}, to maximize their profit, taking wages and productivity as given.

1.3 Carbon taxes and transition risk

The government charges a linear carbon tax from firms on their emissions, which is rebated lump-sum to households as the government balances its budget. The carbon tax rate, τ_t, varies over time, which gives rise to a risk often denoted as transition risk.

1.4 Aggregate emission and physical risk

Since emission arises from each intermediate-goods producer via (4), aggregate emission is

$$X_t = \int X_{it}\, di = A_t \int f_i L_{it}\, di = A_t\, x_t. \tag{6}$$

where x_t is defined by the last equality. We denote x_t as the "detrended" emission since the productivity A_t drives a trend growth in emission X_t, but not in x_t.

Aggregate emission affects the climate, which in turn affects the economy's productivity, A_t. Specifically, productivity grows as

$$dA_t = A_t \mu\, dt + A_t \sigma\, dB_t - A_t x_t \phi_t\, dt. \tag{7}$$

To understand this productivity growth, note that μ is the "normal" technological growth in the absence of climate problems. Further, B_t is a standard Brownian motion that captures the randomness in innovations or technology shocks, and $\sigma > 0$ captures the volatility of these shocks. The last term is the climate element as it captures the notion that a higher aggregate emission lowers productivity, since emissions lead to more destructive weather events and other disruptions.

The process $\phi_t > 0$ quantifies the severity of these climate issues. The climate severity varies over time, creating physical risk, which can be correlated with transition risk. In particular, the carbon tax and climate sensitivity are jointly a finite, time-homogeneous Markov chain denoted by $z_t = (\tau_t, \phi_t)$. The carbon tax can take the values $\tau^1 < \tau^2 < ... < \tau^N$ and the climate severity has support $\phi^1 < \phi^2 < ... < \phi^K$. Thus, from the point of view of consumers and investors, z_t is exogenous and switches from a value $z = (\tau, \phi)$ to another $z' = (\tau', \phi')$ with a constant intensity given by $\lambda_{zz'}$. In other words, over a small time period Δt, this switch occurs with probability $\lambda_{zz'}\Delta t$. Hence, ϕ_t and τ_t can change both separately and simultaneously. For example, a jump in climate severity, ϕ_t, can increase the chance of a policy response in the form of a higher carbon tax, τ_t.

We later show that, in equilibrium, detrended emissions, x_t, only depend on the carbon tax rate. Therefore, we can solve for the productivity as[2]

$$A_t = A_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma B_t - \int_{s=0}^{t} \phi_s x_s \, ds\right). \tag{8}$$

This expression shows explicitly that the cumulative effects of all past emissions, $\int_{s=0}^{t} \phi_s x_s \, ds$, contribute to lower current productivity.

1.5 Asset prices and returns

The endogenous price of the final-goods-producing firm is denoted by V_t and the price of any intermediate goods producer is V_{it}. Corresponding to these asset prices, the instantaneous returns are given by dR_t and dR_{it}, which are the sum of the price appreciation and profits,

[2]We use Itô's Lemma for $\log(A_t)$, integrate, and convert back to A_t.

paid as dividends. For example, the latter return is defined as

$$dR_{it} \equiv \frac{dV_{it} + \Pi_{it} dt}{V_{it}}. \tag{9}$$

The economy also features a risk-free asset paying interest rate r_t. This money-market asset is in zero net supply.

1.6 Households

The representative household has recursive preferences as specified by Duffie and Epstein (1992), with an intertemporal elasticity of substitution (IES) equal to unity. Specifically, the consumption stream $(C_s)_{s \geq t}$ yields a utility level U_t given by

$$U_t = E_t \left[\int_t^{\infty} h(C_s, U_s)\, ds \right], \tag{10}$$

where the utility flow each period is given by

$$h\left(C_t, U_t\right) = \beta(1-\gamma) U_t \left(\log\left(C_t\right) - (1-\gamma)^{-1} \log\left((1-\gamma) U_t\right) \right). \tag{11}$$

Here, $\beta > 0$ is a subjective discount rate and $\gamma > 1$ is the risk-aversion coefficient. We require that the risk aversion be higher than unity, which is necessary (but not sufficient) for a higher future consumption growth to translate into a lower marginal utility when the IES equals one.

The household supplies all of its labor, L, for wage earnings $\mathcal{W}_t L$, and receives a government transfer equal to the sum of all carbon taxes. The household chooses its consumption C_t at each time and its investments in firms and risk-free money-market account to maximize its utility. The household's wealth, W_t, evolves as

$$dW_t = W_t \left(\theta_t\, dR_t + \int_0^1 \theta_{it}\, dR_{it} + \theta_{rt} r_t\, dt \right) + \mathcal{W}_t L\, dt + G_t\, dt - C_t\, dt. \tag{12}$$

Here, θ_t is the fraction of wealth invested in the final-goods firm, θ_{it} is the portfolio weight in each intermediate-good firm, θ_{rt} is the risk-free investment, and these portfolio weights add up to unity, $\theta_t + \int \theta_{it} di + \theta_{rt} = 1$. Further, $G_t = \int \tau_t X_{it}\, di$ is the government transfer, which balances the budget. In other words, the household's wealth grows via its financial returns, wage income, government tax rebates, and is reduced by consumption. The household is originally endowed with full ownership of all the firms. To avoid doubling strategies, we impose the standard constraint $W_t \geq 0$.

1.7 Definition of equilibrium

An equilibrium consists of a collection of intermediate-good prices, p_{it}; labor quantities, L_{it}; a wage rate, $\mathcal{W}_t$; a financial portfolio, $(\theta_t, \theta_{it}, \theta_{rt})$; firm values, V_{it} and V_t; and an interest rate, r_t, such that: (i) all firms maximize profits; (ii) the household maximizes its utility; (iii) the labor market, the markets for intermediate goods, and the markets for financial assets clear.

2 Equilibrium

This section shows how to construct the equilibrium (Sections 2.1), showcasing the model's tractability, and summarizes its main properties in a few propositions (Section 2.2). Further, we present the socially optimal outcome, which also allows us to define the social cost of carbon in the context of our model (Section 2.3).

2.1 Constructing the Equilibrium

Labor choice, wage, prices, and profits (micro)

The final-good firm's profit (23) has first-order condition with respect to intermediate good i given by

$$p_{it} = Y_{it}^{\alpha-1} Y_t^{1-\alpha}. \tag{13}$$

This relation determines the demand curve for good i as

$$Y_{it} = {p_{it}}^{-\frac{1}{1-\alpha}} Y_t, \tag{14}$$

which naturally decreases in price. Based on this demand curve, the intermediate-good firm i chooses its price p_{it}, or equivalently its labor input L_{it}, to maximize its profit

$$\begin{aligned}\Pi_{it} &= p_{it}Y_{it} - \tau_t f_i Y_{it} - \mathcal{W}_t L_{it} = Y_{it}^{\alpha} Y_t^{1-\alpha} - \tau_t f_i Y_{it} - \mathcal{W}_t L_{it} \\ &= A_t^{\alpha} Y_t^{1-\alpha} L_{it}^{\alpha} - \tau_t f_i A_t L_{it} - w_t A_t L_{it},\end{aligned} \tag{15}$$

where we define w_t as the detrended wage, that is, $\mathcal{W}_t = A_t w_t$. We note that many economic variables have as trend the productivity growth, A_t, and we denote the detrended variable by using lowercase.

The optimal labor choice follows as

$$L_{it} = \left(\frac{\alpha}{\tau_t f_i + w_t}\right)^{\frac{1}{1-\alpha}} \frac{Y_t}{A_t} = \left(\frac{\alpha}{\tau_t f_i + w_t}\right)^{\frac{1}{1-\alpha}} \left(\int_0^1 L_{jt}^{\alpha}\, dj\right)^{\frac{1}{\alpha}}. \tag{16}$$

Raising the last equation to power α and integrating across all firms determines the detrended wage level w_t:

$$1 = \int_0^1 \left(\frac{\alpha}{\tau_t f_i + w_t}\right)^{\frac{\alpha}{1-\alpha}} di. \tag{17}$$

We note that w_t depends only on the tax rate τ_t, independently of time, so we use the notation $w_t = w_{\tau_t}$.

To determine labor choices, and thus the level of the output, we equate labor supply, L, with labor demand, which is given by the integral of (16):

$$L = \frac{Y_t}{A_t} \int_0^1 \left(\frac{\alpha}{\tau_t f_i + w_{\tau_t}}\right)^{\frac{1}{1-\alpha}} di, \tag{18}$$

yielding

$$\frac{Y_t}{A_t} = \left(\int_0^1 \left(\frac{\alpha}{\tau_t f_i + w_{\tau_t}} \right)^{\frac{1}{1-\alpha}} di \right)^{-1} L. \tag{19}$$

Combining this result with (16), the labor demand firm of i is seen to be

$$L_{it} = \frac{(\tau_t f_i + w_t)^{\frac{1}{\alpha-1}}}{\int_0^1 (\tau_t f_j + w_t)^{\frac{1}{\alpha-1}} dj} L\,. \tag{20}$$

We see that cleaner firms, that is, firms with lower fossil intensity f_i, employ a larger share of the labor force, and this effect is stronger if the carbon tax is higher. We also see that the labor choice, $L_{it} = L_{i\tau_t}$, only depends on the carbon tax regime, not on time, just as the detrended wage.

Turning to profits, the intermediate good firm i has a profit, which can be calculated using (15) and (16) to be

$$\Pi_{it} = A_t^{\alpha} Y_t^{1-\alpha} L_{it}^{\alpha} - \tau_t f_i A_t L_{it} - w_t A_t L_{it} = (1-\alpha) \left(\frac{\alpha}{\tau_t f_i + w_t} \right)^{\frac{\alpha}{1-\alpha}} Y_t = A_t \pi_{it}, \tag{21}$$

where the detrended profit, π_{it}, is defined through the last equality. Using equation (17), we see that the aggregate profit of all intermediate-good firms is proportional to output:

$$\int_0^1 \Pi_{it}\, di = (1-\alpha) Y_t \int_0^1 \left(\frac{\alpha}{\tau_t f_i + w_t} \right)^{\frac{\alpha}{1-\alpha}} di = (1-\alpha) Y_t. \tag{22}$$

Finally, the final-good firm makes zero profits:

$$\Pi_t = \left(\int Y_{it}^{\alpha}\, di \right)^{\frac{1}{\alpha}} - \int_0^1 p_{it} Y_{it}\, di = Y_t - Y_t^{1-\alpha} \int Y_{it}^{\alpha}\, di = Y_t - Y_t = 0, \tag{23}$$

where the second equality uses (13).

Final-good output, consumption, and emissions (macro)

Aggregate consumption, C_t, equals the output of the final good, Y_t, which is found by integrating the equilibrium intermediate good outputs:

$$C_t = Y_t = \left(\int Y_{it}^{\alpha} di \right)^{\frac{1}{\alpha}} = A_t \left(\int L_{i\tau_t}^{\alpha} di \right)^{\frac{1}{\alpha}} = A_t e^{c_{\tau_t}}, \tag{24}$$

where $c_t = \log\left(\frac{C_t}{A_t}\right)$ is the detrended log-consumption as seen from the last equality.

Aggregate emissions are given by

$$X_t = A_t \int f_i L_{i\tau_t} \, di = A_t \, x_{\tau_t} \,, \tag{25}$$

where x_{τ_t} is the detrended component. Hence, productivity, A_t, evolves based on (7). We can therefore compute the dynamics of the aggregate output and consumption given in (24) using Ito's lemma,

$$\frac{dC_t}{C_t} = \frac{dA_t}{A_t} + e^{\Delta c_{\tau_t}} - 1 = g_{z_t} \, dt + \sigma \, dB_t + e^{\Delta c_{\tau_t}} - 1, \tag{26}$$

where $g_{z_t} = \mu - \phi_t x_{\tau_t}$ is the productivity growth from (7), which depends on the state variable $z_t = (\tau_t, \phi_t)$, that is, it depends on both the carbon tax, τ_t, and the climate sensitivity, ϕ_t. Naturally, aggregate consumption inherits the productivity factor's expected growth rate, which includes the effect of environmental damage, and the exposure to Brownian shocks of the productivity factor, dB_t. The last term, $e^{\Delta c_{\tau_t}} - 1$, captures the fact that aggregate consumption jumps whenever the carbon tax rate changes.

Utility

Finally, we need to derive the utility function (10). We define implicitly the detrended utility process u_t such that the utility, U_t, is written as

$$U_t = \frac{1}{1-\gamma} A_t^{1-\gamma} e^{(1-\gamma) u_t}. \tag{27}$$

We show in the appendix that $u_t = u_{z_t}$ only depends on the state variable $z_t = (\tau_t, \phi_t)$, so we need to find the coefficient in each of the $N \times K$ states, corresponding to N possible values of τ_t and K possible values of ϕ_t. The $N \times K$ unknowns, u_z, can be found as the unique solutions to the $N \times K$ equations

$$u_z = c_z + \frac{g_z}{\beta} + \sum_{z'} \lambda_{zz'} \frac{e^{(1-\gamma)(u_{z'}-u_z)} - 1}{\beta(1-\gamma)} - \frac{\gamma\sigma^2}{2\beta}. \tag{28}$$

The value function coefficient, u_z, consists of three terms: First, the current consumption level c_z plus the discounted integrated future expected incremental consumption due to the current growth, $\frac{g_z}{\beta}$. This term determines utility in a risk-less world, $\lambda =$ and $\sigma = 0$. Second, the summation term, which accounts for jumps to future utility when the state variable changes. Finally, the last term, the constant $-\frac{\gamma\sigma^2}{2\beta}$, accounts for the cost of Brownian risk.

2.2 Equilibrium existence, uniqueness, and properties

We first provide a simple sufficient condition for the equilibrium to exists. Further, the equilibrium is always unique.

Proposition 1 (Existence and uniqueness) *If the subset of firms with zero emissions, $f_i = 0$, has strictly positive mass, then an equilibrium exists and is unique.*

We also record some intuitive properties of the equilibrium.

Proposition 2 (Level-growth tradeoff) *At time t, a higher carbon tax rate, τ_t, leads to lower equilibrium total carbon emissions, X_t, a lower current aggregate consumption, C_t, and a higher consumption growth, g_t.*

This proposition contains several intuitive results. First, a higher carbon tax naturally leads to lower pollution as production is directed toward cleaner firms. Indeed, cleaner firms are less affected by the carbon tax, so they become more profitable, and therefore hire a larger fraction of the labor force and increase production.

When the labor is directed toward cleaner firms, aggregate output falls as production is distorted away from what is *currently* the most productive allocation. However, production *growth* increases — the reduction in pollution pays off over time. Specifically, productivity growth increases as the climate destruction is reduced as seen in (8). In sum, a higher carbon tax reduces near-term consumption, but increases long-term consumption.

2.3 The social cost of carbon: Planner problem

So far, we have considered the competitive equilibrium with time-varying carbon taxes set by an exogenous political process. It is interesting to compare this equilibrium to the socially optimal outcome. Further, understanding the social optimum also allows us to compute the so-called "social cost of carbon" in the context of our model, which is helpful for the purposes of calibrating the model.

The socially optimal outcome solves the planner's problem of maximizing utility taking into account the economy's technology constraints and the effect of pollution:

$$U_t^* = \sup_{\{L_{is}\}_{is}} \mathrm{E}_t \left[\int_t^\infty h(C_s, U_s^*)\, ds \right], \tag{29}$$

where $C_t = A_t \left(\int L_{it}^\alpha \, di \right)^{\frac{1}{\alpha}}$, the dynamics of A_t are in (6)–(7), and $\int L_{it} di \leq L$. This planner problem has a simple and intuitive solution, as we show next.

Proposition 3 (Optimal carbon tax) *The socially optimal labor allocation L_{it}^* depends only on the climate sensitivity, ϕ_t, so that we can write $L_{it}^* = L_{i\phi_t}^*$. It maximizes log-consumption plus scaled growth, $c_t + \frac{g_t}{\beta}$:*

$$\left(L_{i\phi_t}^* \right)_{i \in [0,1]} = \underset{(L_i)_{i\in[0,1]}}{\operatorname{argmax}} \left\{ \frac{1}{\alpha} \log \left(\int L_i^\alpha \, di \right) - \frac{\phi_t}{\beta} \int f_i L_i \, di \right\}. \tag{30}$$

Furthermore, the social optimum can be implemented via the carbon tax

$$\tau^*(\phi_t) = \frac{\alpha \phi_t}{\beta} \left(\int (L_{it}^*)^\alpha \, di \right)^{\frac{1}{\alpha}} = \alpha \times S_t. \tag{31}$$

Here, S_t is the "social cost of carbon," defined as the planner's Lagrange multiplier with respect to the aggregate emission constraint (6), denoted by η_t, divided by marginal utility: $S_t = \eta_t / h_C(C_t^*, U_t^*)$.

This proposition contains several natural results.

First, the optimal labor allocations in this model are exclusively a function of the current climate sensitivity ϕ_t.

Second, the social optimum is found by trading off the utility impacts of the current labor allocation on the current versus future levels of consumption. The former is simply the current log-consumption level, c_t, while the latter is given by the cumulated present value of the future-consumption effect of the current growth rate, g_t. Since changes to consumption are permanent, this second effect equals $\frac{g_t}{\beta}$.

Third, the planner solution can be implemented as a market equilibrium with a carbon tax given by (31). The optimal carbon tax balances the cost and benefit of a high carbon tax as in the social optimum: A high tax (i) lowers current consumption, c_t; and (ii) increases consumption growth, $\frac{g_t}{\beta}$, due to the emission externality.

Fourth, the optimal carbon tax is a constant fraction, α, of the social cost of carbon, S_t, which captures the disutility to the central planner from increasing emissions by a unit, expressed in dollar units. While the optimal tax equals the social cost of carbon in standard models with perfect competition, there is a small difference in here due to the market power of any firm i, where market power depends on how much α is below one.

3 Results on Climate Risk Pricing

We next turn to the model's asset pricing implications.

3.1 State prices

The representative investor consumes the entire output of the final good in equilibrium. Given this consumption process, the agent derives utility U_t as specified in (10). The agent's marginal rate of substitution process, which is a stochastic discount factor (SDF) in this economy, is well known (see, for instance, Duffie and Epstein (1992)) to be given by

$$M_t = h_C(C_t, U_t) e^{\int_0^t h_U(C_s, U_s) ds}, \tag{32}$$

where we recall that h is the utility flow (11), while h_C and h_U are its partial derivatives. Using the rewriting (27), the log-SDF can be expressed as

$$\log M_t = -\gamma \log A_t + \beta \int_0^t \left((1-\gamma)\left(c_s - u_s\right) - 1 \right) ds + \log m_{z_t}, \tag{33}$$

where

$$\log m_{z_t} \equiv -c_{z_t} - (\gamma - 1) u_{z_t} \tag{34}$$

is the detrended SDF, which captures the market price of transition risk and physical risk.

Based on this expression, the following proposition states our fundamental observation that the SDF is U-shaped in the carbon tax.

Proposition 4 (SDF and carbon taxes) *(i) For every ϕ, there exists a value $\bar{\tau}(\phi)$ such that the detrended SDF, m_z, decreases in τ for $\tau < \bar{\tau}(\phi)$ and increases in τ for $\tau \geq \bar{\tau}(\phi)$ keeping fixed ϕ, for $\|\lambda\|$ small enough. Furthermore, the critical tax level $\bar{\tau}(\phi)$ increases in climate severity, ϕ, and the risk aversion, γ. (ii) The detrended SDF m_z increases with ϕ, keeping τ fixed, for $\|\lambda\|$ small enough.*

An example of a U-shaped SDF is illustrated in Figure 1(b) in the introduction. The U-shaped SDF implies that an increase in the carbon tax from a low starting point lowers the SDF—in other words, the higher tax corresponds to a better state of the world.

The general result in Proposition 4 is predicated on changes in tax rates being rare, which makes the current utility and marginal utility levels primarily determined by the current tax rate and growth sensitivity.[3] Numerical examples such as the calibration in Figure 1(b) illustrate that the conclusion of the proposition holds for values of λ that we think empirically relevant.

This result is based on equations (28) and (34), which imply that the logarithm of marginal utility is a negative multiple of $\beta^{-1} g + \frac{\gamma}{\gamma - 1} c$ (up to an additive constant). We

[3] This condition can be relaxed, but some restriction is necessary. Consider, for instance, the opposite extreme, when a change in tax rate is imminent. In that case, current utility does not depend on the current consumption level, yet marginal utility does; consequently, as $\|\lambda\|$ grows without bound, m increases at all values $\tau > 0$. Naturally, this is not an interesting case, as it renders current policy irrelevant.

have already observed the natural outcome that the tax rate has opposite impacts on these two terms: a higher tax rate increases the growth rate, but decreases the current output. Furthermore, it turns out that, taken as decreasing functions of each other as τ changes, g and c are concave, at least for small λ, which explains the proposition.

Finally, from our discussion at the end of Section 2.1 it follows that the household's (detrended) utility, given by u, is also determined by a combination of the consumption growth rate and the current consumption level: $\beta^{-1}g + c$. The same conclusions as for the marginal utility can therefore be drawn, with the opposite sign, for the level of the utility. Furthermore, the critical value for the utility level is lower than the one for the marginal utility, $\bar{\tau}(\phi)$.

3.2 Asset prices

Having determined an SDF for our economy, we can use it to price all traded assets, in particular the equity of intermediate-good firms. Specifically,

$$V_{it} = \mathrm{E}_t \left[\int_t^\infty \frac{M_s}{M_t} \Pi_{is} \, ds \right] = A_t \mathrm{E}_t \left[\int_t^\infty \frac{M_s}{M_t} \frac{A_s}{A_t} \pi_{is} \, ds \right]. \tag{35}$$

Since the dynamics of M and A depend only on the Markov state z_t, the expectation on the right-hand side of equation (35) is only a function of z_t. We can therefore write

$$V_{it} = A_t \, v_{iz_t}. \tag{36}$$

Based on these asset prices, we next consider how prices change when the carbon tax changes. Naturally, such prices changes are different for green and brown firms, where we refer to firm g as greener than firm b whenever $f_g < f_b$.

Proposition 5 (Transition risk and contemporaneous returns) *If the carbon tax increases as the state changes from $z = (\tau, \phi)$ to $z' = (\tau', \phi')$, thus $\tau' > \tau$, then the price of a browner asset b will drop relative to that of a greener asset g, that is, $\frac{v_{gz'}}{v_{gz}} > \frac{v_{bz'}}{v_{bz}}$, if $\|\lambda\|$ is sufficiently small.*

The result that a carbon tax increase leads to a drop in brown firm values relative to green ones is illustrated in Figure 1(a) in the introduction.

Based on the asset prices (36), the return on stock i can be written as

$$dR_{it} = \frac{dV_{it} + \Pi_{it}dt}{V_{it}} = \frac{dA_t}{A_t} + \frac{\pi_{iz_t}}{v_{iz_t}}dt + \frac{v_{iz_t} - v_{iz_{t-}}}{v_{iz_{t-}}}, \tag{37}$$

where the last term captures the return due to a state change at time t. We have the following result.

Proposition 6 (Green security market line) *When the economy is in state z, the equilibrium expected excess return of any asset i is*

$$\frac{1}{dt}E_t^z\left[dR_{it} - r_z\right] = \gamma\sigma^2 + \sum_{z'} \lambda_{zz'} \left(1 - \frac{m_{z'}}{m_z}\right)\left(\frac{v_{iz'}}{v_{iz}} - 1\right) \tag{38}$$

and the risk-free rate is given by

$$r_z = \beta + \beta(\gamma - 1)\left(c_z - u_z\right) + \gamma g_z - \gamma(\gamma+1)\frac{\sigma^2}{2} + \sum_{z'} \lambda_{zz'}\left(1 - e^{c_z - c_{z'} + (\gamma-1)(u_z - u_{z'})}\right). \tag{39}$$

Equation (38) is a specialization of the standard result that expected excess returns equal the (negative of the) covariance of the returns and the SDF increment. The first term on the right-hand side represents the covariance of the Brownian component of the returns, A_t, with the Brownian component of the SDF, $A_t^{-\gamma}$. The second term, the summation, represents the covariance of the jump components of the two processes. A negative covariance, which increases the expected excess return, simply means that the firm value jumps up when marginal utility falls. The next section shows how the green security market line depends on firms' emission intensities.

3.3 Greenium driven by transition and physical risks

This section studies the greenium, that is, the required return of green assets relative to brown. If the current tax rate is low (relative to the critical level $\bar{\tau}$ identified in Proposition 4), then an increase in the tax rate corresponds to a lower marginal utility state, as long as the increase is not too high and it is not accompanied by a large increase in the sensitivity ϕ. So we get a positive greenium under the assumption that τ_t and ϕ_t have "independent jump times," meaning that the jump intensity from any state $z = (\tau, \phi)$ to $z' = (\tau', \phi')$ is zero, $\lambda_{zz'} = 0$, unless either (i) only ϕ jumps (i.e., $\tau = \tau'$), or (ii) only τ jumps ($\phi = \phi'$). We also need the assumption that τ has "local jumps," meaning that $\lambda_{zz'} = 0$ unless τ and τ' differ by one notch (e.g., when the tax is $\tau_t = \tau^n$, it can only jump to τ^{n-1} or τ^{n+1}). We have:

Proposition 7 (Greenium with independent risks) *Provided that τ_t and ϕ_t have independent jump times, τ has local jumps, and $\|\lambda\|$ is sufficiently low, the expected return of brown firms is lower than that of green firms when the tax rate is low and higher when the tax rate is high.*

The proposition confirms the intuition that brown firms have lower expected returns when taxes are too low — as may be the case currently in a large part of the world — provided that regulatory and physical risks are unrelated. This result is illustrated in the lower line Figure 2 in the introduction.

It is plausible, however, that physical risk contributes to regulatory risk. In particular, that higher physical risk, that is, increases in ϕ, precipitate the adoption of policies amounting to increases in τ. We capture this possibility parsimoniously, albeit perhaps a little starkly, by allowing for joint jumps in ϕ and τ.[4]

In other words, it is interesting to consider what happens to the greenium when ϕ_t and τ_t have more correlated jumps. To address this question, we need to first define what it means to have more correlated jumps. We say that τ_t and ϕ_t have stronger jump dependence if the intensity of jumping from any state $z = (\tau, \phi)$ to $z' = (\tau', \phi')$ satisfies that (i) $\lambda_{zz'}$ weakly increases for jumps in the same direction, $(\tau' - \tau)(\phi' - \phi) > 0$; (ii) $\lambda_{zz'}$ weakly decreases for jumps in the opposite direction, $(\tau' - \tau)(\phi' - \phi) < 0$; and (iii) marginal jump intensities remain unchanged.[5]

[4] An alternative specification with similar economic implications would have the jump in ϕ only lead to a very high intensity of a jump in τ. Our assumption is computationally convenient and allows for a simple formulation of Proposition 8, in particular for the assumption of a sufficiently small λ.

[5] While τ and ϕ are not individually Markovian, we can still define their marginal jump intensities at each state as follows. Given the state z, a jump from τ to any τ' arrives with intensity $\lambda^{\tau}_{z,\tau'} := \sum_{\phi'} \lambda_{z,(\tau',\phi')}$. We define $\lambda^{\phi}_{z,\phi'}$ analogously, and denote λ^{τ} and λ^{ϕ} as the marginal jump intensities.

Proposition 8 (Greenium with related risks) *If τ has local jumps then, for $\|\lambda\|$ sufficiently small, the return greenium at t decreases if τ_t and ϕ_t have stronger jump dependence.*

This result is illustrated in Figure 2 in the introduction. As seen as in the figure, when the risks have stronger dependence (upper line), the required return–emission relation is less negative compared to the independent case (lower line).

3.4 Portfolio tilts

When all households are identical, prices and expected returns adjust so that everyone holds the market portfolio in equilibrium. However, if investors differ in their climate beliefs, who will hold the greener portfolio? In other words, if investors invest based on risk considerations, who will drive the greenium in the right direction?

To address this question, we next consider investors who differ in their climate beliefs. To capture different beliefs about how the environment affects the economy, recall that productivity, A_t, depends on emission via the sensitivity ϕ in (7). In the interest of transparency, we assume that ϕ is constant. Investors differ in their views on the value of ϕ, with investor j holding the belief $\phi_{(j)}$. For example, an investor with $\phi_{(j)} < \phi$ is a climate-skeptic, that is, someone who thinks that the climate has little effect on the economy. In contrast, a climate-concerned investor with $\phi_{(j)} > \phi$ believes that the climate has a large effect. Finally, rational investors use $\phi_{(j)} = \phi$, and, to focus here on portfolio choice, we assume that these investors set the prices as in Section 3.2.[6]

To see investors' portfolio choices easily, suppose that there are only two types of firms, "brown" and "green." Green firms have emissions rate of $f_i = f_g$ while brown firms have higher emission rates of $f_i = f_b$, where $f_b > f_g$. Any firm $i \in [0, m_g]$ is green, where $m_g \in (0, 1)$ is the ratio of green firms, and the rest are brown.

A portfolio is characterized by the fraction of wealth invested in brown firms, green firms, final-good firms, and risk-free securities. Since final-good firms are worth zero, we disregard these, and let $\theta = (\theta_g, \theta_b)^\top$ be the portfolio of green and brown investments such that the

[6] Rational prices are ensured by having unit mass of representative households using the true ϕ and a range of atomistic agents with different beliefs $\phi_{(j)}$. Note that we assume that agents' beliefs are dogmatic, meaning that they are not adjusted as output realizes over time.

rest goes into the risk-free asset, $\theta_r = 1 - \theta_g - \theta_b$.

The market portfolio is denoted as $\theta^m = (\theta_g^m, \theta_b^m)^\top$, and we let $\theta^{GMB} = (1, -1)^\top$ be the green-minus-brown (GMB) portfolio. The GMB portfolio is often used in the empirical literature to capture the return difference between green and brown stocks.

Any stock portfolio θ can be seen as a combination of the market portfolio and the GMB portfolio. In other words, we can always write a portfolio as

$$\theta = \omega^m \theta^m + \omega^{GMB} \theta^{GMB} \tag{40}$$

for $\omega^m, \omega^{GMB} \in \mathbb{R}$. We say that a portfolio θ is "tilted towards green stocks" if $\omega^{GMB} > 0$.

The key question is what type of investors are tilted toward green stocks? The following proposition provides an answer under the technical condition that the jumps in the tax rate are not too large. To implement this condition, we define the tax in the n-th tax regime as $\tau^n(\rho) = \hat{\tau} + (\hat{\tau}^n - \hat{\tau})\rho$, where $\rho \in (0, 1]$, $0 \leq \hat{\tau}$, and $0 \leq \hat{\tau}^1 < \ldots < \hat{\tau}^{N_\tau}$. We have the following result.

Proposition 9 (Bet on the other team) *As long as $\|\lambda\|$ and ρ are small enough, a climate-skeptic agent with beliefs given by $\phi_{(j)} < \phi$ tilts her portfolio towards green stocks. Conversely, a climate-concerned agent with $\phi_{(j)} > \phi$ tilts her portfolio towards brown stocks, as long as $\phi_{(j)}$ is not too large.*

This proposition delivers a surprising result as real-world climate-concerned investors are often assumed to make green investments. We show that, from a risk-management perspective, they might in fact do the opposite. To understand this result, note that investors with different climate beliefs, $\phi_{(j)}$, differ along two dimensions.

First, investors differ in their views on the overall economic growth. Climate skeptics are less worried that pollution will reduce productivity, so they are most optimistic about growth. These differences of beliefs about overall economic growth lead to different investment in the market portfolio.

Second, investors differ in their views on the effect of a jump in the carbon tax rate. Different views on jumps in the carbon tax rate lead to different investments in the GMB portfolio. Indeed, green and brown stocks have the same exposure to the Brownian shock dB_t, but different exposures to tax jumps.[7]

A climate-skeptic views such an increase in the tax rate as a relatively bad state of nature, because a high carbon tax reduces near-term consumption and the climate skeptic does not believe in the long-term benefits. Therefore, the climate skeptic wants to hedge this risk, which is achieved by tilting her portfolio towards assets that do well when taxes increase, i.e., green stocks.

In contrast climate-concerned investors are worried that carbon taxes go down (or rise slower than expected), so they tilt their portfolio towards assets that do well when carbon taxes decrease, i.e., brown stocks.

Just like sports fans sometimes bet on the opposing team to hedge their risk — either their team wins or their bet pays off — so do investors in our model.

4 Calibration

This section calibrates the model to study the magnitudes of the model-implied effects of climate risk pricing. We first choose specific parameter values and then compute the equilibrium and study its properties.

4.1 Parameter values

Starting with households' preferences, we choose a risk aversion parameter $\gamma = 6$ and a time preference rate of $\beta = 0.03$.

Turning to the firms, we consider two types of firms, green (g) and brown (b). Firms $i \in [0, 0.9]$ are green and the rest $i \in (0.9, 1]$ are brown, meaning that 90% of firms are green. Greens have lower emission intensities than brown, $f_g < f_b$. Specifically, the emission

[7] As seen in the proof Proposition 9, the reactions of value functions and the returns are linear in the tax-rate jumps to the first order with respect to tax-rate jump sizes. So, effectively, there are only two risks in the economy — the Brownian shock dB_t and the arrival of a jump $\Delta\tau_t$. As a consequence, if the tax-rate jumps are small enough, the two stocks along with the risk-free asset (approximately) complete the market.

intensities are chosen such that

$$\frac{f_b}{f_g} = 51 \tag{41}$$

$$0.9 f_g + 0.1 f_b = 4.2 \times 10^{-4}\ tCO_2/\$. \tag{42}$$

Condition (41) means that, with zero taxes, brown firms account for $\frac{.1\times 51}{.1\times 51 + .9\times 1} = 85\%$ of total emissions. This specification captures that highly skewed distribution of real-world emission in which about 10% of firms account for the bulk of emissions.

Condition (42) means that the real-world total emission to output (right-hand side) equals the same quantity in the model when the tax is zero (left-hand side). Specifically, the right-hand side is obtained by dividing 2024 world emissions by 2024 world GDP.

The production-function parameter α controls the profit share of the intermediate firms, thus profits as a share of GDP. More important for our purposes, it also determines the degree of substitutability between green and brown firms; the higher α the less inefficient it is to shift production from brown to green firms. We choose $\alpha = 0.8$, a typical value of this parameter in the macro-finance literature. The zero-emission growth rate of output is $\mu = 0.03$, while its diffusive standard deviation is $\sigma = 0.08$, in between the consumption-growth volatility and dividend volatility in the data. Given the risk aversion $\gamma = 6$, this latter parameter generates a risk premium of around $\gamma\sigma^2 = 3.84\%$.

Climate risk is given by the distribution of physical risk ϕ_t and transition risk τ_t. There are $K = 3$ levels of the physical risk, $\phi^1 < \phi^2 < \phi^3$. To set the median value, ϕ^2, we rely on equation (31), which ties it to the social cost of capital. Specifically, we set it so that the social cost of carbon is \$200/$\mathrm{tCO_2}$ when $\tau = 0$, motivated by the \$190/$\mathrm{tCO_2}$ estimate from EPA (2023). Given the discount rate of $\beta = 0.03$, this results in a value $\phi^2 = \beta \times S = 6$. We set the other values of physical risk symmetrically around the median, $\phi^1 = 1.5$, and $\phi^3 = 10.5$.

For the carbon tax, τ_t, we consider a symmetric distribution with $N = 33$ equally spaced values, $\tau^1 = 0$, $\tau^2 = 10$, ..., $\tau^{33} = 320$. The median value in state 17 is a carbon tax of $\tau^{17} = 160$, which is the optimal tax given by Proposition 3 when ϕ equals its median value of 6.

Finally, we choose the jump intensities for $z_t = (\tau_t, \phi_t)$ by first specifying the marginal jump intensities for τ_t and ϕ_t, respectively, and then their correlation. For simplicity, we assume that τ_t and ϕ_t can only jump one state up or down.

Recall the notation that $\lambda^\phi_{1,2}$ is the intensity of ϕ_t jumping from state 1 to state 2. With this notation, we set $\lambda^\phi_{1,2} = \lambda^\phi_{3,2} = 0.1$, meaning that physical risk jumps toward its median value in state 2 about once per 10 years. We set the intensity of jumping away from state 2 to be slightly lower, $\lambda^\phi_{2,1} = \lambda^\phi_{2,3} = 0.075$.

We set the intensity of transitioning towards the median, thus $\lambda^\tau_{n,n+1}$ for $n < 17$ and $\lambda^\tau_{n,n-1}$ for $n > 17$, to the value $\bar{\lambda}^\tau = 0.5$. We further set the intensities of transitioning away from the median to $0.2 \times \bar{\lambda}^\tau = 0.1$. We can think of small, relatively frequent changes to the tax rates as carbon-tax increases in different jurisdictions.[8]

The jump-correlation for τ_t and ϕ_t is captured by a parameter denoted by $\varrho \in [0, 1]$. When $\varrho = 0$, jumps are independent and almost surely never happen at the same time. More generally, ϱ helps determine the joint transition intensity λ, which determines the probability of (τ_t, ϕ_t) jumping from any state (n, k) to any other nearby state. Using the notation $\iota \in \{-1, 1\}$ to denote whether we consider up-jumps or down-jumps, the joint intensity is[9]

$$\lambda_{(n,k),(n+\iota,k+\iota)} = \varrho \min\{\lambda^\tau_{n,n+\iota}, \lambda^\phi_{k,k+\iota}\} \tag{44}$$

$$\lambda_{(n,k),(n+\iota,k)} = \lambda^\tau_{n,n+\iota} - \lambda_{(n,k),(n+\iota,k+\iota)} \tag{45}$$

$$\lambda_{(n,k),(n,k+\iota)} = \lambda^\phi_{k,k+\iota} - \lambda_{(n,k),(n+\iota,k+\iota)}. \tag{46}$$

We see from (44) how ϱ determine the intensity of common jumps. We exclude simultaneous jumps to τ and ϕ going in opposite directions. The independent jumps specified in (45) and (46) are compensated such that the marginal jump intensities remain λ^τ and λ^ϕ.

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[8]For a sense of what our choice of $\bar{\lambda}^\tau$ means for long-term carbon taxes, we can compute

$$\mathrm{E}\left[\tau_{t=20} \,|\, \tau_0 = \tau^2\right] = 90.1. \tag{43}$$

In words, the expected tax rate after 20 years is approximately 90. It takes about 60 years for this expected rate to come within one percent of the expected optimal value.

[9]Clearly, the intensity is well defined only both states (n, k) and $(n + \iota, k + \iota)$ are in the support, which means that $1 \le n, n + \iota \le N$ and $1 \le k, k + \iota \le K$.

4.2 Calibrated SDF, returns, and greenium

Figures 1 and 2 in the introduction calibrate the model's SDF, realized returns, and required returns based on the parameters in Section 4.1. We explain each figure in turn.

Figure 1(b) plots the SDF. Specifically, we assume that the economy is currently, at time 0, in state $(\tau_0, \phi_0) = (10, 6)$. The figure plots the SDF if the carbon tax jumps to a new level τ, normalized by the current SDF, that is, $m_{\tau,\phi_0}/m_{\tau_0,\phi_0}$ for a range of values of τ.

In a similar spirit, to contruct Figure 1(a), we first consider the valuation of green and brown firms at time 0. Based on (36), we know that $V_{it} = A_t\, v_{iz_t}$, and compute v_{gz_t} and v_{bz_t} for each state $z_t = (\tau_t, \phi_t)$ using Proposition 6. Hence, if the tax rate jumps to a new level, τ, then the realized return of green firms will be $\frac{v_{g,(\tau,\phi_0)}}{v_{g,(\tau_0,\phi_0)}}$ and similarly for brown firms.[10] This realized return is plotted in Figure 1(a) for each value of τ.

To generate Figure 2, we first compute the expected return of green and brown stocks, respectively, using Proposition 6. To illustrate expected returns for firms with a broader range of emission intensities, we also consider non-atomistic firms characterized by emission intensities, f_j, ranging between $f_g = 7 \times 10^{-5}$ and 0.008, which is higher than $f_b = 0.0036$. We compute these expected returns in two scenarios: with independent jumps ($\varrho = 0$), depicted as the lower line, and with dependent jumps ($\varrho = 1$), shown as upper line.

4.3 Calibrated portfolios of climate concerned vs. skeptics

We illustrate numerically the results of Section 3.4 by considering a range of investor beliefs $\phi_{(j)}$, namely $[0, 12]$, while the correct value is $\phi = 6$. By solving exactly the agent's dynamic problem we obtain her portfolio, which we project on the market portfolio and the GMB portfolio as given by equation (40). Figure 3 plots the GMB loading, ω^{GMB}.

As predicted by Proposition 9, climate skeptics, who hold beliefs $\phi_{(j)} < \phi$, deviate from the market portfolio by tilting towards green stocks: their loading ω^{GMB} is positive. Conversely, a belief $\phi_{(j)} > \phi$ translates into a shunning of green stocks. Consistent with the small return differential between green and brown stocks, portfolio tilts are small.

[10]Our calibration allows only for local jumps in τ; returns associated with larger jumps, thus $n > 3$ given that $\tau_0 = \tau^2$, occur with probability zero.

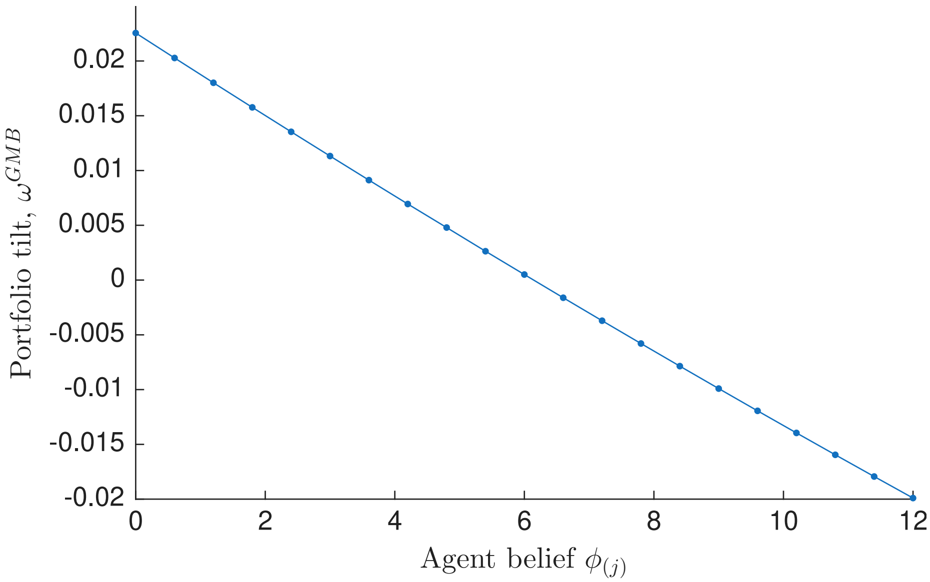

Figure 3: **Green-minus-brown portfolio tilts.** This figure shows the portfolio tilt toward green stocks (y-axis) for agents characterised by their belief, $\phi_{(j)}$, in how much emissions affect the climate (x-axis). The agent on the left believes that emissions have no effect on the climate, and this agent buys green stocks to hedge against overregulation. The agent on the right believes that emissions have large effects, and this agent buys brown stocks to hedge against climate problems due to too low carbon taxes. The portfolio tilt is defined in equation (40) and model parameters are in Section 4.1.

4.4 Sensitivity analyses

We consider here the way in which some of the more important parameterization choices we made impact our greenium results. One such choice is the distribution of future carbon taxes, which we vary in this section by considering a range of values for the parameter $\bar{\lambda}^{\tau}$, which determines the marginal transition intensities for τ_t.

Specifically, we let $\bar{\lambda}^{\tau}$ take values ranging from 0.1 to 10 times that of the baseline value of 0.5 and compute the expected returns given the current state (τ_0, ϕ_0) for both green and brown firms. Figure 4 plots these values. We notice in particular the non-monotonic pattern of returns, most easily visible for brown firms (R_b^e).

The explanation lies with two opposite roles played by the transition intensities. On the one hand, higher transition intensities reduce the utility, and therefore marginal utility, impact of the current state. This effect leads to a lower greenium, since higher taxes are associated with lower decreases, perhaps even increases, in marginal utility. On the other hand, higher transition intensities make the realization of the transition risk more frequent,

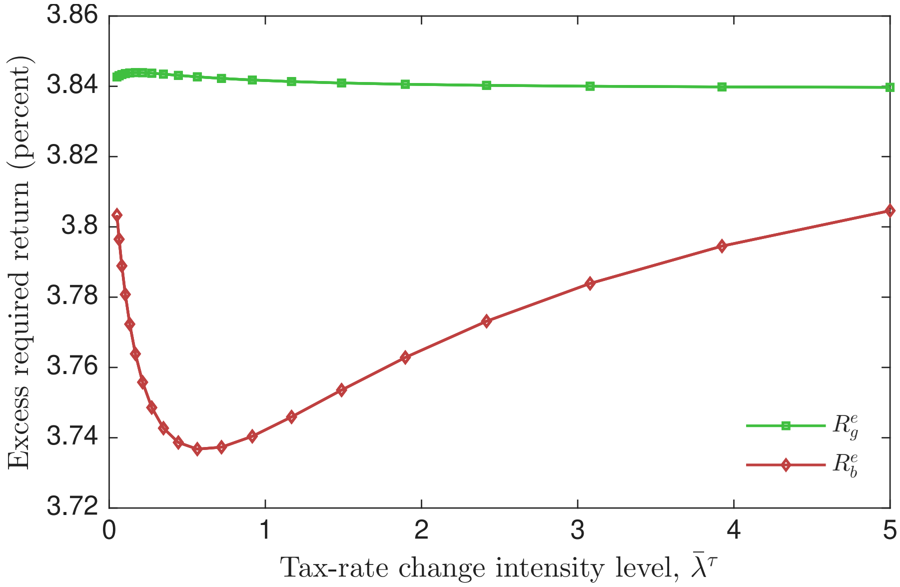

Figure 4: **Sensitivity to tax-change intensity.** This figure shows the excess required returns for green and brown firms, respectively, for a range of values of the scaling parameter $\bar{\lambda}^{\tau}$ governing the transition intensities λ^{τ}. Parameter values and further details are provided in Section 4.1.

hence give it a higher weight in the determination of expected returns. Indeed, in the trivial case $\bar{\lambda}^{\tau} = 0$ the greenium is zero. This effect pushes the greenium up further into positive territory.

Another parameter for which a relatively wide range of values might be justifiable is the substitutability parameter α. On the one hand, based on typical markup estimates, a value $\alpha = 0.85$ appears most reasonable. On the other hand, macro growth models using the Dixit-Stiglitz framework typically use estimates for the elasticity in the range 1.5–4, which translate, in our notation, to a range 0.33–0.8 for the parameter α.

We therefore compute, for a broad range of values for α, the expected returns to green and brown stocks, and plot the results in Figure 5. We notice that the expected-return effect is virtually zero for small α, but grows at an increasing rate as the elasticity of substitution $(1-\alpha)^{-1}$ becomes very high. This pattern is the natural consequence of the effect of substitutability on the cash-flow impact of the tax rate.

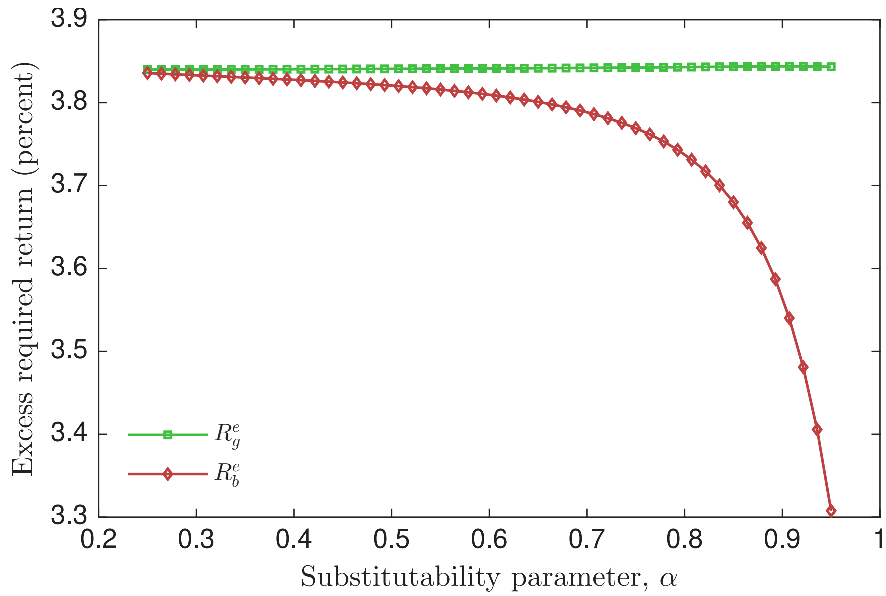

Figure 5: **Sensitivity to substitutability.** This figure shows the excess required returns for green and brown firms, respectively, for a range of values of the parameter α governing intermediate goods' substitutability in the production of the final good, and consequently the firms' profit share. All model parameters are in Section 4.1.

5 Conclusion

Climate risk is one of the major risks facing society as a whole and each investor individually. The societal best response is to introduce an optimal carbon tax, but how should investors respond when the actual tax is far below this level?

This paper develops a tractable environmental macro-finance model in which higher carbon taxes shift production from brown to green firms, reduce current output, and improve future growth by slowing climate damages. When taxes start from a low level, a tax increase is welfare-improving, so climate risk is less about losses when regulation tightens and more about the risk that regulation remains too weak for too long.

In such an environment, brown firms can hedge climate risk: Their payoffs are relatively high in states with intensifying climate damages following low carbon taxes, making their required returns lower than those of green firms. This positive greenium obtains when tax changes are largely political, so that transition and physical risks do not interact.

In contrast, when higher carbon tax result from negative climate news, higher taxes can coincide with bad news overall, flattening or even reversing the return-emissions relation. A quantitative calibration shows that the greenium arising purely from climate risk pricing can take either sign and is small in magnitude, typically below one percentage point per year in absolute value across parameterizations.

We also find surprising results on portfolio choice across investors who differ in their climate beliefs. Climate-concerned investors optimally "bet on the other team" by tilting toward brown firms, which pay off in the scenarios they fear most. In a similar spirit, climate skeptics tilt toward green firms that pay off in high-tax scenarios that they view as costly over-regulation.

In sum, societal management of climate risk cannot be delegated via individual risk management. Indeed, individual risk management can rationally go in the opposite direction, supporting brown firms as hedges against climate-policy failure.

References

Acharya, V. V., R. Berner, R. Engle, H. Jung, J. Stroebel, X. Zeng, and Y. Zhao (2023). Climate stress testing. *Annual Review of Financial Economics 15*(1), 291–326.

Allen, F., A. Barbalau, and F. Zeni (2023). Reducing carbon using regulatory and financial market tools. *Working paper, Imperial College London*.

Ardia, D., K. Bluteau, K. Boudt, and K. Inghelbrecht (2023). Climate change concerns and the performance of green vs. brown stocks. *Management Science 69*(12), 7607–7632.

Baker, S. D., B. Hollifield, and E. Osambela (2022). Asset prices and portfolios with externalities. *Review of Finance 26*(6), 1433–1468.

Baker, S. R., N. Bloom, and S. J. Davis (2016). Measuring economic policy uncertainty. *The quarterly journal of economics 131*(4), 1593–1636.

Bansal, R., D. Kiku, and M. Ochoa (2020). Climate change and growth risks. *Working Paper, Duke University*.

Bansal, R., D. Kiku, and M. Ochoa (2021). Price of long-run temperature shifts in capital markets. *Working Paper, Duke University*.

Barnett, M., W. Brock, and L. P. Hansen (2020). Pricing uncertainty induced by climate change. *The Review of Financial Studies 33*(3), 1024–1066.

Barnett, M., W. A. Brock, H. Zhang, and L. P. Hansen (2024). Uncertainty, social valuation, and climate change. *University of Chicago, Becker Friedman Institute for Economics Working Paper* (2024-75).

Bauer, R., K. Gödker, P. Smeets, and F. Zimmermann (2024). Mental models in financial markets: How do experts reason about the pricing of climate risk? *Working paper, Maastricht University*.

Bolton, P. and M. Kacperczyk (2021). Do investors care about carbon risk? *Journal of financial economics 142*(2), 517–549.

Bolton, P. and M. Kacperczyk (2023). Global pricing of carbon-transition risk. *The Journal of Finance 78*(6), 3677–3754.

Bonnefon, J.-F., A. Landier, P. Sastry, and D. Thesmar (2025). The moral preferences of investors: Experimental evidence. *Journal of Financial Economics 163*, 103955.

Chikhani, P. and J. Renne (2025). An analytical framework to price long-dated climate-exposed assets. *Quantitative Economics 16*(4), 1093–1146.

Daniel, K. D., R. B. Litterman, and G. Wagner (2019). Declining co2 price paths. *Proceedings of the National Academy of Sciences 116*(42), 20886–20891.

Duffie, D. and L. G. Epstein (1992). Stochastic differential utility. *Econometrica 60*(2), 353–394.

Edmans, A., T. Gosling, and D. Jenter (2024). Sustainable investing: Evidence from the field. *FEB-RN Research Paper* (18).

Engle, R. F., S. Giglio, B. Kelly, H. Lee, and J. Stroebel (2020). Hedging climate change news. *The Review of Financial Studies 33*(3), 1184–1216.

EPA (2023). EPA report on the social cost of greenhouse gases: Estimates incorporating recent scientific advances.

Eskildsen, M., M. Ibert, T. I. Jensen, and L. H. Pedersen (2024). In search of the true greenium. *Working paper, Copenhagen Business School*.

Faccini, R., R. Matin, and G. Skiadopoulos (2023). Dissecting climate risks: Are they reflected in stock prices? *Journal of Banking & Finance 155*, 106948.

Giglio, S., B. Kelly, and J. Stroebel (2021a). Climate finance. *Annual Review of Financial Economics 13*(1), 15–36.

Giglio, S., M. Maggiori, K. Rao, J. Stroebel, and A. Weber (2021b). Climate change and long-run discount rates: Evidence from real estate. *The Review of Financial Studies 34*(8), 3527–3571.

Giglio, S., M. Maggiori, J. Stroebel, Z. Tan, S. Utkus, and X. Xu (2025). Four facts about ESG beliefs and investor portfolios. *Journal of Financial Economics 164*, 103984.

Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica 82*(1), 41–88.

Heeb, F., J. F. Kölbel, S. Ramelli, and A. Vasileva (2023). Sustainable investing and political behavior. *Working Paper, MIT*.

Hong, H., N. Wang, and J. Yang (2023a). Mitigating disaster risks in the age of climate change. *Econometrica 91*(5), 1763–1802.

Hong, H., N. Wang, and J. Yang (2023b). Welfare consequences of sustainable finance. *Review of Financial Studies 36*, 4864–4918.

Huynh, T. D. and Y. Xia (2021). Climate change news risk and corporate bond returns. *Journal of Financial and Quantitative Analysis 56*(6), 1985–2009.

Kelly, B., L. Pástor, and P. Veronesi (2016). The price of political uncertainty: Theory and evidence from the option market. *The Journal of Finance 71*(5), 2417–2480.

Krueger, P., Z. Sautner, and L. T. Starks (2020). The importance of climate risks for institutional investors. *The Review of financial studies 33*(3), 1067–1111.

Network for Greening the Financial System (2020). Overview of environmental risk analysis by financial institutions.

Nobel Committee (2018). Economic growth, technological change, and climate change. *Scientific Background on the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2018*.

Nordhaus, W. (2019). Climate change: The ultimate challenge for economics. *American Economic Review 109*(6), 1991–2014.

Nordhaus, W. D. (1994). *Managing the global commons: the economics of climate change*, Volume 31. MIT press Cambridge, MA.

Pástor, L., R. F. Stambaugh, and L. A. Taylor (2021). Sustainable investing in equilibrium. *Journal of Financial Economics 142*(2), 550–571.

Pástor, L., R. F. Stambaugh, and L. A. Taylor (2022). Dissecting green returns. *Journal of Financial Economics 146*(2), 403–424.

Pástor, L., R. F. Stambaugh, and L. A. Taylor (2025a). Carbon burden.

Pástor, L., R. F. Stambaugh, and L. A. Taylor (2025b). Sustainable investing. *Annual Review of Financial Economics 17*.

Peck, S. C. and T. J. Teisberg (1992). Ceta: a model for carbon emissions trajectory assessment. *The Energy Journal 13*(1).

Pedersen, L. H. (2023). Carbon pricing versus green finance. *Working paper, Copenhagen Business School*.

Pedersen, L. H., S. Fitzgibbons, and L. Pomorski (2021). Responsible investing: The ESG-efficient frontier. *Journal of Financial Economics 142*(2), 572–597.

Sautner, Z., L. Van Lent, G. Vilkov, and R. Zhang (2023). Firm-level climate change exposure. *The Journal of Finance 78*(3), 1449–1498.

Seltzer, L. H., L. Starks, and Q. Zhu (2022). Climate regulatory risk and corporate bonds. Technical report, National Bureau of Economic Research.

Stroebel, J. and J. Wurgler (2021). What do you think about climate finance? *Journal of Financial Economics 142*(2), 487–498.

Zhang, S. (2023). Carbon Returns Across the Globe. *Journal of Finance (forthcoming)*.

A Appendix

Proof of Proposition 1. We conjecture and verify the following utility function

$$U_t = \frac{1}{1-\gamma} A_t^{1-\gamma} e^{(1-\gamma)u_{z_t}}. \tag{A.1}$$

Using this expression and the following implication of the definition (10) of utility,

$$\frac{1}{dt}\mathrm{E}_t[dU_t] + h(C_t, U_t) = 0\,, \tag{A.2}$$

we get the equation

$$u_z = -\frac{\gamma\sigma^2}{2\beta} + \frac{g_z}{\beta} + c_z + \sum_{z'} \lambda_{zz'} \frac{e^{(1-\gamma)(u_{z'}-u_z)} - 1}{\beta(1-\gamma)}. \tag{A.3}$$

We note that the left-hand side (LHS) of (A.3) increases from $-\infty$ to ∞ with u_z, while the right-hand side (RHS) decreases in u_z; consequently, fixing $u_{z'}$ for all $z' \neq z$, a (unique) solution of (A.3) exists for u_z. Furthermore, there exists a value $\bar{M} < \infty$ such that, if $u_z < \bar{M}$ for all $z' \neq z$, then $u_z < \bar{M}$. Indeed, in (A.3) the LHS is higher than the RHS at $u_z = \bar{M}$ as long as

$$\bar{M} \geq -\frac{\gamma\sigma^2}{2\beta} + \frac{g_z}{\beta} + c_z. \tag{A.4}$$

Using a similar reasoning for a lower bound $\underline{M}$, we conclude that (A.3) defines a continuous mapping from $[\underline{M}, \bar{M}]^N$ into itself, and therefore has a fixed point by Brouwer's Fixed Point Theorem.

For uniqueness, consider two different solutions to the system (A.3), u^1 and u^2 with $u_z^1 > u_z^2$ for at least one value of z, and let $\Delta u_z = u_z^1 - u_z^2$. By subtracting from each other (A.3) for the two solutions, we obtain

$$\Delta u_z = \sum_{z'} \lambda'_{zz'} \frac{e^{(1-\gamma)(\Delta u_{z'}-\Delta u_z)} - 1}{\beta(1-\gamma)} \tag{A.5}$$

with $\lambda'_{zz'} = \lambda_{zz'} e^{(1-\gamma)(u_{z'}^2 - u_z^2)} > 0$ whenever $\lambda_{zz'} > 0$. We therefore have

$$\Delta u_z \leq \sum_{z'} \lambda'_{zz'} \left(\Delta u_{z'} - \Delta u_z\right). \tag{A.6}$$

Isolating Δu_z, we therefore have

$$\Delta u_z \leq \frac{\sum_{z'} \lambda'_{zz'}}{1 + \sum_{z'} \lambda'_{zz'}} \Delta u_{z'} < \max_{z'} \Delta u_{z'}, \tag{A.7}$$

since by assumption $\max_{z'} \Delta u_{z'} > 0$. We therefore obtain the contradiction $\max_z \Delta u_z < \max_z \Delta u_z$, showing that we cannot have $u^1 \neq u^2$. $\square$

Proof of Proposition 2. The proofs of the statements of the proposition are contained in the proof of Proposition 4. $\square$

Proof of Proposition 3. We conjecture a central-planner value function depending only on the current level of productivity, A_t, and the state z_t, which here is synonymous with ϕ_t. Given the parametric specification of the aggregator function h, our conjecture is

$$U^*(A, z) = \frac{1}{1-\gamma} A^{1-\gamma} e^{(1-\gamma)u_z}. \tag{A.8}$$

The Hamilton-Jacobi-Bellman (HJB) equation under this conjecture gives

$$\begin{aligned} 0 = &\sup_{\{L_i^*\}_i} h(A_t e^{c_t}, U^*(A_t, z_t)) + \frac{dU^*(A_t, z_t)}{dA} A_t g + \frac{1}{2} \frac{d^2 U^*(A_t, z_t)}{dA^2} A_t^2 \sigma^2 \\ &\quad + U^*(A_t, z_t) \sum_{z'} \lambda_{zz'} \left(e^{(1-\gamma)(u_{z'} - u_z)} - 1 \right) \qquad \text{(A.9)} \\ = &\sup_{\{L_i^*\}_i} (1-\gamma) U^*(A_t, z_t)(\beta c + g) + (1-\gamma) U^*(A_t, z_t) \left(-\frac{\gamma}{2} \sigma^2 \right) \\ &\quad - \beta(1-\gamma) U^*(A_t, z_t) u_z + U^*(A_t, z_t) \sum_{z'} \lambda_{zz'} \left(e^{(1-\gamma)(u_{z'} - u_z)} - 1 \right). \qquad \text{(A.10)} \end{aligned}$$

We see that the choice of the labor allocation must be made to maximize $\beta c + g$ because nothing else depends on L_i, as stated in the proposition. Furthermore, a solution to this system of algebraic equations in u_z exists as shown in Proposition 1, so that a function of the form (A.8) that solves the HJB equation exists. The verification argument does not pose any difficulty.

We now show that $\{L_i^*\}_i$ can be implemented with a tax rate. We start with the planner's problem, that is, maximizing $c + g/\beta$ subject to $\int L_i \, di \leq L$, with Lagrangean

$$\frac{1}{\alpha} \log \left(\int L_i^\alpha \, di \right) - \frac{\phi}{\beta} \int f_i L_i \, di - \nu \left(\int L_i \, di - L \right), \tag{A.11}$$

where the first term captures log-consumption, the second term the part of the consumption growth that is affected by pollution, and the third term has Lagrange multiplier ν. The solution is

$$L_i^* = \left(\frac{\phi}{\beta} f_i + \nu\right)^{\frac{1}{\alpha-1}} \left(\int (L_j^*)^\alpha \, dj\right)^{\frac{1}{\alpha-1}}. \tag{A.12}$$

Equation (16), on the other hand, is the market-determined labor allocation

$$L_i = \left(\frac{\tau_t f_i + w_t}{\alpha}\right)^{\frac{1}{\alpha-1}} \left(\int_0^1 L_j^\alpha \, dj\right)^{\frac{1}{\alpha}}. \tag{A.13}$$

Equations (A.12) and (A.13) are identical as long as

$$\frac{\tau_t}{\alpha} = \frac{\phi}{\beta} \left(\int (L_j^*)^\alpha \, dj\right)^{\frac{1}{\alpha}} \tag{A.14}$$

$$\frac{w_t}{\alpha} = \nu \left(\int (L_j^*)^\alpha \, dj\right)^{\frac{1}{\alpha}}. \tag{A.15}$$

The first condition is satisfied by setting τ_t appropriately, i.e.,

$$\tau^* = \frac{\alpha\phi}{\beta} \left(\int (L_i^*)^\alpha \, di\right)^{\frac{1}{\alpha}} = \frac{\alpha\phi}{\beta} \frac{Y^*}{A^*}. \tag{A.16}$$

For the second, we note that, with $\tau_t = \tau^*$, if w_t happened to satisfy (A.15) then the labor market in the decentralized economy clears if and only if the labor-market constraint is satisfied (with equality) in the central-planning economy. Thus, w_t given by (A.15) is a decentralized-market outcome, and $L_i = L_i^*$.

The final statement to prove concerns the social cost of carbon. We defined it in term of the Lagrangian η_t in the planner's problem in which we do not substitute directly the dependence of aggregate emissions on the labor choices:

$$\sup_{\{L_{is}, X_s, \eta_s\}_{is}} \mathrm{E}_t \left[\int_t^\infty h(C_s, U_s^*) \, ds - \int_t^\infty \eta_s \left(A_s \int f_i L_{is} \, di - X_s\right) ds\right] \tag{A.17}$$

subject to

$$dA_t = A_t\left(\mu\, dt + \sigma dB_t\right) - \phi X_t\, dt \tag{A.18}$$

$$C_t = A_t\left(\int_0^1 L_{it}^{\alpha} di\right)^{\frac{1}{\alpha}} \tag{A.19}$$

$$\int L_i\, di \leq L. \tag{A.20}$$

Rewriting the HJB equation yields

$$\begin{aligned}0 = \sup_{\{L_i^*\}_i, x, \eta}\ & (1-\gamma)U^*(A_t, z_t)(\beta c + \mu - \phi x) + (1-\gamma)U^*(A_t, z_t)\left(-\frac{\gamma}{2}\sigma^2\right) \\ & - \beta(1-\gamma)U^*(A_t, z_t)u_z + U^*(A_t, z_t)\sum_{z'}\lambda_{zz'}\left(e^{(1-\gamma)(u_{z'}-u_z)} - 1\right) \\ & - \eta A_t\left(\int f_i L_i\, di - x\right).\end{aligned} \tag{A.21}$$

The FOC with respect to scaled emissions x is

$$\eta A_t = (1-\gamma)U^*(A_t, z_t)\phi \tag{A.22}$$

and we also have, given the specification of h,

$$h_C\left(C^*, U^*\right) = \beta(1-\gamma)\frac{U^*}{C^*}. \tag{A.23}$$

It follows that

$$S_t \equiv \frac{\eta_t}{h_C\left(C_t^*, U_t^*\right)} = \frac{\phi}{\beta}\frac{C_t^*}{A_t} = \frac{\phi}{\beta}\left(\int L_i^{\alpha}\, di\right)^{\frac{1}{\alpha}}. \tag{A.24}$$

□

Proof of Proposition 4. We start by showing that, fixing ϕ, (a) g increases strictly with τ and (b) c decreases strictly with τ. Furthermore, given (a) and (*(ii)*)b), there is a one-to-one relation between $c(\tau)$ and $g(\tau)$ and we show that (c) $c \circ g^{-1}$ is a strictly concave function. These properties are independent of the value of λ. We conclude that, when $\lambda = 0$, $m_{\tau^n} = -c_{\tau^n} - (\gamma-1)u_{\tau^n} = -\gamma c_{\tau^n} - (\gamma-1)\frac{g_{\tau^n}}{\beta}$ is U-shaped in τ. This conclusion also holds in a neighborhood of $\lambda = 0$, by continuity of the functions $\lambda \mapsto c_z + (\gamma-1)u_z$ and the fact that the state space is finite.

Our objects of interest, g and c, are functions of τ given by

$$g = \mu - \phi \int f_i L_i \, di \tag{A.25}$$

$$c = \frac{1}{\alpha} \log \left(\int L_i^{\alpha} \, di \right), \tag{A.26}$$

where L_i and w are themselves functions of τ that solve (20) and (17), i.e.,

$$L_i = (\tau f_i + w)^{-\frac{1}{1-\alpha}} \left(\int (\tau f_j + w)^{-\frac{1}{1-\alpha}} \, dj \right)^{-1} L \tag{A.27}$$

$$\int (\tau f_i + w)^{-\frac{\alpha}{1-\alpha}} \, di = \alpha^{-\frac{\alpha}{1-\alpha}}. \tag{A.28}$$

To simplify derivations, we let $\bar{w}(\tau) = \frac{w(\tau)}{\tau}$ and set $L = 1$ without loss of generality. We rewrite (A.27) as

$$L_i = (f_i + \bar{w})^{-\frac{1}{1-\alpha}} \left(\int (f_j + \bar{w})^{-\frac{1}{1-\alpha}} \, dj \right)^{-1} \tag{A.29}$$

We note that $\bar{w}$ is monotonic in τ, specifically (A.28) shows that $\bar{w}$ is strictly decreasing in τ. We can therefore consider the properties of g and c as functions of $\bar{w}$, and we proceed to prove the proposition based on this idea.

Before proceeding to computing their derivatives, we rewrite g as

$$\begin{aligned} g &= \mu - \phi \int f_i (f_i + \bar{w})^{-\frac{1}{1-\alpha}} \, di \left(\int (f_j + \bar{w})^{-\frac{1}{1-\alpha}} \, dj \right)^{-1} \\ &= \mu - \phi \left(\int (f_i + \bar{w})^{-\frac{1}{1-\alpha}+1} \, di - \bar{w} \int (f_i + \bar{w})^{-\frac{1}{1-\alpha}} \, di \right) \left(\int (f_j + \bar{w})^{-\frac{1}{1-\alpha}} \, dj \right)^{-1} \\ &= \mu - \phi \int (f_i + \bar{w})^{-\frac{1}{1-\alpha}+1} \, di \left(\int (f_j + \bar{w})^{-\frac{1}{1-\alpha}} \, dj \right)^{-1} + \phi \bar{w}. \end{aligned} \tag{A.30}$$

We further rewrite g as

$$g = \mu - \phi \Xi_q + \phi \bar{w} \tag{A.31}$$

where $q = \frac{1}{1-\alpha} > 1$ and, for any q',

$$\Upsilon_{q'} = \int (f_i + \bar{w})^{-q'} \, di \tag{A.32}$$

$$\Xi_{q'} = \frac{\Upsilon_{q'-1}}{\Upsilon_{q'}}. \tag{A.33}$$

We can compute the derivatives of g and c with respect to $\bar{w}$:

$$\begin{aligned} \phi^{-1} \frac{dg}{d\bar{w}} &= 1 - (1-q) - q \Upsilon_{q-1} \Upsilon_{q+1} \Upsilon_q^{-2} \\ &= q \left(1 - \frac{\Xi_q}{\Xi_{q+1}} \right). \end{aligned} \tag{A.34}$$

For clarity we record the following short result, which follows immediately by applying the Cauchy-Schwarz inequality.

Lemma 1 *For any q', $\Xi_{q'+1} < \Xi_{q'}$.*

Applying Lemma (1), we see that $\frac{dg}{d\bar{w}} < 0$, so that $\frac{dg}{d\tau} > 0$, thus establishing (a). We also have

$$c = \frac{1}{\alpha} \log(\Upsilon_{q\alpha}) - \log(\Upsilon_q) = \frac{1}{\alpha} \log(\Upsilon_{q-1}) - \log(\Upsilon_q) \tag{A.35}$$

$$\frac{dc}{d\bar{w}} = -q \frac{\Upsilon_q}{\Upsilon_{q-1}} + q \frac{\Upsilon_{q+1}}{\Upsilon_q} = q \left(\Xi_q - \Xi_{q+1} \right). \tag{A.36}$$

Again using Lemma (1), we deduce $\frac{dc}{d\bar{w}} > 0$, establishing (b).

We can furthermore compute the slope of the curve (g, c) traced by varying τ as

$$\frac{dc}{dg} = \frac{dc}{d\bar{w}} \left(\frac{dg}{d\bar{w}} \right)^{-1} = -\Xi_q, \tag{A.37}$$

so that

$$\frac{d}{d\bar{w}} \frac{dc}{dg} = q \frac{\Upsilon_{q+1}}{\Upsilon_{q-1}} - (q-1) \frac{\Upsilon_q^2}{\Upsilon_{q-1}^2} = q \left(\Xi_{q+1}^{-1} - \Xi_q^{-1} \right) \Xi_q^{-1} + \Xi_q^{-2} > 0, \tag{A.38}$$

using again Lemma (1). Since $\frac{dg}{d\bar{w}} < 0$, we have concluded that $\frac{d^2c}{dg^2} < 0$, which is statement (c).

Consequently, the log-marginal utility

$$\log m = -c - (\gamma - 1)u = -\gamma c - \beta^{-1}(\gamma - 1)g \tag{A.39}$$

is strictly convex in g (where, again, we can think of it as a function of g rather than τ since g is monotonic in τ). Thus, m decreases in g if and only if g is lower than a critical value. Since g increases strictly with τ, a critical value $\bar{\tau}$ as stated by the proposition exists.

Furthermore, when $\lambda = 0$, this critical value is determined by $\gamma c'(\bar{\tau}) = -(\gamma - 1)\frac{g'(\bar{\tau})}{\beta} = (\gamma - 1)\phi\frac{x'(\bar{\tau})}{\beta}$. A higher ϕ therefore decreases the marginal impact $-\gamma c'(\bar{\tau}) - (\gamma - 1)\frac{g'(\bar{\tau})}{\beta}$, making it negative. Since $\log m$ is U-shaped, its minimum value must now lie to the right of $\bar{\tau}$.

As for risk aversion, we can apply the same reasoning replacing ϕ with the ratio $\frac{\gamma-1}{\gamma}$, which increases in γ.

For part *(ii)* of the proposition, it suffices to note that g decreases with ϕ together with equation (A.39). □

Proof of Proposition 5. We start by showing that if $f_i < f_j$ and $\tau < \tau'$ then

$$\frac{\pi_{i\tau'}}{\pi_{i\tau}} > \frac{\pi_{j\tau'}}{\pi_{j\tau}}. \tag{A.40}$$

We have from (21) that

$$\frac{\pi_{i\tau}}{\pi_{j\tau}} = \left(\frac{f_j + \frac{w_\tau}{\tau}}{f_i + \frac{w_\tau}{\tau}}\right)^{\frac{\alpha}{1-\alpha}}, \tag{A.41}$$

so we need that $\frac{w_\tau}{\tau}$ decreases with τ, which we already shown in the proof of Proposition 4 (see text after equation (A.29)).

In the limit $\lambda = 0$, price-to-dividend ratios are constant and equal to the consumption-wealth ratio in the economy, which is β due to unitary IES, proving the result in this limiting case. By continuity, it also holds for λ close enough to zero. □

Proof of Proposition 6. This proposition states the standard result expressing the excess expected return as the negative of the covariance between return and SDF, and the risk-free return as the reciprocal of the mean of the SDF. □

Proof of Proposition 7. We start by noting that, up to $O(\|\lambda\|)$,

$$v_{i,nl} = \pi_{i,nl}\xi_{nl}, \tag{A.42}$$

where ξ_{nl} is the value at time 0 of receiving A_t/A_0 in perpetuity when A_t grows at constant rate $\mu - \phi_l x_n$. Thus, this valuation ratio is given by the consumption-to-wealth ratio in the economy, which, given the unitary IES, is β and therefore independent of n and l. Furthermore, $\pi_{i,nl}$ does not depend on the growth rate ϕ_l, thus on l.

It follows that, up to $O(\|\lambda\|)$, $v_{i,nl} = \beta^{-1}\pi_{i,n}$, the same as in the economy without physical risk. We simplify by writing $v_{i,n}$ instead. As a consequence, only changes to τ — jumps from n to k — have a return effect.

The proposition then follows immediately from Propositions 5 and 6. In particular, equation (38) implies, using the notation m_{nl} for the detrended SDF when $\tau = \tau^n$ and $\phi = \phi^l$,

$$\frac{1}{dt}\mathrm{E}_t^n\left[dR_{it} - dR_{jt}\right] = \gamma\sigma^2 + \sum_{z'} \lambda_{zz'}\left(1 - \frac{m_{z'}}{m_z}\right)\left(\frac{v_{iz'}}{v_{iz}} - \frac{v_{jz'}}{v_{jz}}\right), \tag{A.43}$$

and we have (i) for $\tau^{n+1} < \bar{\tau}$, $\frac{m_{n+1l}}{m_{nl}} < 1$ and (ii) $\frac{v_{i,n+1}}{v_{i,n}} > \frac{v_{j,n+1}}{v_{j,n}}$ if $f_i < f_j$. $\square$

Proof of Proposition 8. Up to terms of order $O(\|\lambda\|)$, we get from (28) that

$$u_{n,l} = \frac{\mu}{\beta} - \frac{\gamma\sigma^2}{2\beta} - \frac{\phi^l}{\beta}X_n + c_n, \tag{A.44}$$

which decreases with ϕ. As before (equation (34)), we have

$$\log m_{nl} = -c_n - (\gamma - 1)u_{nl}, \tag{A.45}$$

an increasing function of l.

As we noted in the proof of Proposition 7, to the leading order $v_{i,nl} = v_{i,n}$ is independent of l. The GMB premium follows as

$$\begin{aligned}
&\sum_{k,j}\lambda_{nl,kj}\left(1 - \frac{m_{kj}}{m_{nl}}\right)\left(\frac{v_{g,k}}{v_{g,n}} - \frac{v_{b,k}}{v_{b,n}}\right) \\
&= \sum_{k,j}\lambda_{nl,kj}\left(1 - \frac{m_{kl}}{m_{nl}}\right)\left(\frac{v_{g,k}}{v_{g,n}} - \frac{v_{b,k}}{v_{b,n}}\right) + \sum_{k,j}\lambda_{nl,kj}\left(\frac{m_{kl}}{m_{nl}} - \frac{m_{kj}}{m_{nl}}\right)\left(\frac{v_{g,k}}{v_{g,n}} - \frac{v_{b,k}}{v_{b,n}}\right) \\
&= \sum_{k}\lambda_{nk}^{\tau}\left(1 - \frac{m_{kl}}{m_{nl}}\right)\left(\frac{v_{g,k}}{v_{g,n}} - \frac{v_{b,k}}{v_{b,n}}\right) - \sum_{k,j}\lambda_{nl,kj}\left(\frac{m_{kj}}{m_{nl}} - \frac{m_{kl}}{m_{nl}}\right)\left(\frac{v_{g,k}}{v_{g,n}} - \frac{v_{b,k}}{v_{b,n}}\right).
\end{aligned} \tag{A.46}$$

In this derivation, the second line obtains from the first by adding and subtracting $\sum_{k,j}\lambda_{nl,kj}\frac{m_{kl}}{m_{nl}}\left(\frac{v_{g,k}}{v_{g,n}} - \frac{v_{b,k}}{v_{b,n}}\right)$ and regrouping terms. The last line uses the definition of λ^{τ} in

Footnote 5 as the intensity, given any state z, of a jump from τ to any τ', $\lambda^{\tau}_{z,\tau'} := \sum_{\phi'} \lambda_{z,(\tau',\phi')}$.

Since, as we noted above, m_{nl} increases with l, we have

$$\frac{m_{kl}}{m_{nl}} < \frac{m_{kj}}{m_{nl}} \tag{A.47}$$

when $j > l$. By the assumption of the proposition, the transition from (n, l) to (k, j) with $j > l$ can only happen if $k \geq n$; symmetrically, if $j < l$ then $k \leq n$. Since, by Proposition 5, $\frac{v_{g,k}}{v_{g,n}} \geq \frac{v_{b,k}}{v_{b,n}}$ when $k \geq n$ and conversely, we conclude that the second summation in (A.46) is positive. □

Proof of Proposition 9. We prove the proposition by analyzing the HJB equation of agent $\phi_{(j)}$. We start by conjecturing the agent's value function, given her wealth W and the current tax rate, to have the form

$$J(W, \tau^n) = \frac{W^{1-\gamma}}{1-\gamma} e^{(1-\gamma)\bar{u}^j_n}, \tag{A.48}$$

where $\{u^j_n\}_n$ are scalars.

All agents agree on the asset prices being given by

$$V_{i,t} = A_t v_{i,\tau_t}, \tag{A.49}$$

even as they do not agree on the expected growth of A_t. We write the return on firm i, as perceived by agent j,

$$dR^j_{i,t} = \frac{dA_t}{A_t} + \frac{\pi_{i,\tau^n}}{v_{i,\tau^n}} dt + \frac{\Delta v_{i,\tau_t}}{v_{i,\tau^n}} = \mu^j \, dt + \sigma dB^j_t + \Delta R_{i,\tau_t}, \tag{A.50}$$

where B^j is a Brownian motion under the beliefs of agent j. We have the HJB equation

$$\begin{aligned} 0 = \sup_{\theta} & -\beta u^j + \bar{K} + \theta^\top \mu^j - \frac{1}{2}\gamma\sigma^2 \left(\theta^\top \mathbf{1}\right)^2 + \\ & \frac{1}{1-\gamma} \sum_k \lambda_{nk} \left(\left(1 + \sum_i \theta_i \Delta R_{i,nk} \right)^{1-\gamma} e^{(1-\gamma)(\bar{u}^j_k - \bar{u}^j_n)} - 1 \right) \end{aligned} \tag{A.51}$$

for a constant $\bar{K}$ independent of beliefs.

The FOC is

$$0 = \mu^j - \gamma\sigma^2 \mathbf{1}\mathbf{1}^\top \theta + \sum_k \lambda_{nk} \Delta R_{nk} \left(1 + \theta^\top \Delta R_{nk}\right)^{-\gamma} e^{(1-\gamma)(\bar{u}^j_k - \bar{u}^j_n)}, \tag{A.52}$$

which to the lowest orders in λ and ρ becomes

$$0 = \mu^j - \gamma\sigma^2 \mathbf{1}\mathbf{1}^\top \theta + \sum_k \lambda_{nk} \Delta R_{nk} \left(1 - \gamma\theta^\top \Delta R_{nk}\right) e^{(1-\gamma)(\bar{u}^j_k - \bar{u}^j_n)}, \tag{A.53}$$

under the conjecture $\Delta R_{i,nk} = S_i \left(\tau^k - \tau^n\right) + o(\rho)$ for a constant S_i depending on the asset. We also conjecture $\bar{u}^j_k - \bar{u}^j_n = T^j \left(\tau^k - \tau^n\right) + o(\rho)$ for a constant T^j depending only on the agent (through her beliefs $\phi_{(j)}$).

We subtract from the equation above the analogous one holding for the representative investor, who holds the market portfolio, to get, with $\delta\theta := \theta^j - \theta^m$,

$$\begin{aligned} 0 = &-(\phi_{(j)} - \phi)x\mathbf{1} - \gamma\sigma^2(\mathbf{1}^\top \delta\theta)\mathbf{1} + \sum_k \lambda_{nk} \Delta R_{nk} (1-\gamma)(\bar{u}^j_k - \bar{u}_k - \bar{u}^j_n + \bar{u}_n) \\ &- \gamma \sum_k \lambda_{nk} \Delta R_{nk} \Delta R_{nk}^\top \delta\theta. \end{aligned} \tag{A.54}$$

Noting that $\mu^j - \mu = -(\phi_{(j)} - \phi)x\mathbf{1}$, it follows that

$$\begin{aligned} 0 = &(\phi - \phi_{(j)})x\mathbf{1} - \gamma\sigma^2 \mathbf{1}\mathbf{1}^\top \delta\theta + \\ &(1-\gamma)S(T^j - T) \sum_k \lambda_{nk} \left(\tau^k - \tau^n\right)^2 - \gamma S \sum_k \lambda_{nk} \left(\tau^k - \tau^n\right)^2 S^\top \delta\theta. \end{aligned} \tag{A.55}$$

We note that the vectors $\mathbf{1}$ and S are linearly independent: the two stock prices do not have identical jumps when the tax rates change. In fact, $S_g > S_b$. Consequently, up to the second order in ρ, the FOCs imply

$$0 = (\phi - \phi_{(j)})x - \gamma\sigma^2 \mathbf{1}^\top \delta\theta \tag{A.56}$$

$$0 = (1-\gamma)(T^j - T) - \gamma S^\top \delta\theta. \tag{A.57}$$

We note now that $\mathbf{1}^\top \theta^m > 0$ and $S^\top \theta^m < 0$.[11] From the previous two equations, we have $\mathbf{1}^\top \delta\theta > 0$, since $\phi_{(j)} < \phi$, and $S^\top \delta\theta > 0$, which follows from $T^j < T$, which we prove below.

[11] This follows from the fact that aggregate financial wealth is a constant proportion of total wealth, which is proportional to output. Consequently, an increase in the tax rate, which decreases consumption, decreases financial wealth — the market-portfolio return is negative.

Given the decomposition

$$\delta\theta = \omega^m \theta^m + \omega^{GMB} \theta^{GMB}, \tag{A.58}$$

taking inner product with $\mathbf{1}$ gives $0 < \omega^m \mathbf{1}^\top \theta^m$, so that $\omega^m > 0$: agent j is more heavily invested in the market than the representative agent. As for the tilt, premultiplication by S gives

$$0 < \omega^m S^\top \theta^m + \omega^{GMB} S^\top \theta^{GMB}, \tag{A.59}$$

which means

$$\omega^{GMB}(S_g - S_b) > -\omega^m S^\top \theta^m > 0. \tag{A.60}$$

We conclude that $\omega^{GMB} > 0$, so that the deviation $\delta\theta$, and thus θ^j, is titled towards green stocks.

It remains to show that $T^j < T$, i.e., the growth-component of the utility of the agent skeptical about the impact of emissions on growth is less sensitive to taxes. This fact can be seen in the agent's HJB equation (A.51), which, for small enough λ, implies

$$\beta u^j \approx \bar{K} + \sup_\theta \left\{ \theta^\top \mu^j - \frac{1}{2}\gamma\sigma^2 (\mathbf{1}^\top \theta)^2 \right\} \tag{A.61}$$

$$= \bar{K} + \frac{1}{2\gamma} \frac{(\bar{\mu} + (\phi - \phi_{(j)})x(\tau))^2}{\sigma^2}, \tag{A.62}$$

where $\bar{\mu} = \mu + \beta - \gamma\sigma^2$ is the expected return on (all) stocks when $\lambda \approx 0$.

It follows that

$$T^j = \frac{du^j}{d\tau} \approx -\frac{x'}{\gamma} \frac{(\bar{\mu} + (\phi - \phi_{(j)})x)}{\sigma^2} \phi_{(j)}. \tag{A.63}$$

We have $x' < 0$. Therefore, as long as $\phi_{(j)}x < \bar{\mu} + \phi x$, T^j increases with $\phi_{(j)}$, so that $T^j - T < 0$.

$\square$